UNIVERSITY OF CALIFORNIA, SAN DIEGO

Essays in Dynamic Uncertainty: Behavioral Economics, Investment Theory and Law and Economics

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Economics by

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The dissertation of Steven E. Scroggin is approved, and it is acceptable in quality and form for publication on microfilm:

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DEDICATION

For Karen

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ABSTRACT OF THE DISSERTATION

Essays in Dynamic Uncertainty: Behavioral Economics, Investment Theory and Law and Economics

by

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Doctor of Philosophy in Economics
University of California, San Diego, 2005
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The dissertation is composed of three papers on three distinct topics. The papers and their abstracts are:

Exploitable Play of Believers in the "Law of Small Numbers" in Repeated Constant-Sum Games: In repeated fixed-pair constant-sum games with unique equilibria in mixed strategies, rational players avoid exploitable play. Play is exploitable if it deviates systematically from the mixed strategy equilibrium choice probabilities, or if current play fails to be serially independent of past play. Experimental subjects often exhibit exploitable patterns. I develop a model to find patterns of serial dependence, to forecast play and detect players trying to exploit their opponent's patterns.

Investment and Cash Flow in Dynamic Firms Facing Uncertainty and Liquidity Constraints: Suppose firms facing conventional production functions, constant returns to scale and symmetric convex adjustment costs take random prices as given and choose capital and labor optimally. In a two-period model, a firm with perfect access to capital markets earns more expected cash flow and expects to invest more after a mean-preserving spread in output prices; a similar liquidity-constrained firm may expect less cash flow and investment after such a change.

Ethics, Economics and Lawyers' Conflicts of Interest: When a lawyer represents more than one client, the effect of common agency on clients depends upon the nature of the strategic interaction between the clients. Common agency can be synergistic, destructive or neutral. A simple game theory approach to the relationship between the principals distinguishes these three situations and shows when common agency is relatively efficient from the clients' perspective. A reputational dynamic imperfectly implements efficiency. Third party enforcement through the institutions of legal discipline can encourage efficient behavior if the legal rules are efficient. I show that the U.S. law regarding lawyers common agency is usually efficient in that the outcomes of the cases are aligned with efficiency. Further, while the frameworks are very different, legal analysis and economic analysis tend to treat similar cases similarly. Where economics and law differ, the analysis may suggest how to make the positive law of lawyers' conflicts of interest more efficient.

ACKNOWLEDGMENT

Modified versions of Chapters 1 and 3 have been submitted for publication. The dissertation author was the sole author of both papers.

Chapter I

Exploitable Play of Believers in the "Law of Small Numbers" in Repeated Constant-Sum Games

ABSTRACT

In repeated fixed-pair constant-sum games with unique equilibria in mixed strategies, rational players avoid exploitable play. Play is exploitable if it deviates systematically from the mixed strategy equilibrium choice probabilities, or if current play fails to be serially independent of past play. Experimental subjects often exhibit exploitable patterns. I develop a model to find these patterns of serial dependence, to forecast play and detect players trying to exploit their opponent's patterns.

Keywords: Behavioral economics, experimental economics, bounded rationality, game theory

JEL Classification: C7, C9

I.A Introduction

In repeated fixed-pair constant-sum games with unique equilibria in mixed strategies, rational players avoid exploitable play. In such games, the only subgame-perfect equilibrium is repeating the stage game mixed equilibrium (Shachat & Wooders (2001)). Play is *exploitable* if it deviates systematically from the mixed strategy equilibrium choice probabilities or if current play fails to be serially independent of past play.

In experiments with such games, subjects' play is sometimes exploitable. Although research has focused mainly on systematic deviations from the mixed strategy equilibrium choice probabilities, there is good reason to believe that failures of serial independence are also important. Cognitive psychologists who study individual decisions (Edwards (1961)) and more recently some who study behavior in repeated games with mixed-strategy equilibria (Rapoport & Budescu (1992)(R&B)) question whether people can produce serially-independent sequences when motivated to do so, and whether they can recognize deviations from serial independence. The effort to model behavioral production and recognition of serial independence has roots in the representativeness heuristic first identified by Kahneman & Tversky (1972). Representativeness is the heuristic of identifying a class with an exemplar. In particular, experimental subjects have displayed a systematic bias known as "local representativeness" in the production and recognition of randomness. Local representativeness subjects exaggerate how likely it is that a small sample resembles the parent population from which it is drawn.

The learning models that have been used to detect deviations from equilibrium mixing in fixed-pair constant sum games are not well suited to detect the kind of failure of serial independence that local representativeness implies. Learning models like fictitious play impose linearity (and so additive separability) on players' responses to past history. By contrast, the usual models of local representativeness take sequences of past play as their arguments and they are not additively separable. Consequently, familiar learning theory models are mis-specified to detect deviations from randomness from a source that appears plausible, given evidence from the cognitive psychology literature.

In this paper, I examine temporal patterns in subjects' play using a model well suited to identify the kinds of serial dependence suggested by local representativeness. I use data from experiments conducted by Mookherjee & Sopher (1994)(M&S) and R&B to identify and anticipate representativeness bias in the data using simple nonlinear additively non-separable dynamic decision rules. The decision rules respect the informational constraints on the players. The rules use observable history to determine when play is forecastable, and then forecast play. Similar rules forecast best responses to forecastable play. One of the decision rules I use is closely related to a rule originally suggested by R&B. Neither R&B nor I claim that these rules are optimal, yet this class of rules can do substantially better than any familiar alternative (including equilibrium play) against hypothetical opponents in the M&S and R&B data sets. I make no attempt in this paper to characterize optimal or equilibrium play by or against players subject to representativeness bias. A theory of optimal or equilibrium play by or against players subject to representativeness bias is the subject of work in progress with Vincent Crawford.

Using data from games rather than a decision environment raises issues which may be of interest to game theorists as well as decision analysts. Players may approach repeated games that have unique equilibria in mixed strategies with a defensive or an offensive posture. A defensive player wants to avoid loss by avoiding exploitable patterns in his paths of play; his aim is to be random and elusive. The ideal defense is of course the serially independent repeated Nash equilibrium strategy, but local representativeness bias may interfere with a defensive player's efforts to produce it[1]. By contrast, an offensive player expends most of his effort on finding a way to win by outwitting his opponent; he scans his enemy's paths in search of a pattern, but representativeness bias may also limit his ability to perceive such patterns. Of course, if he attacks by responding to a perceived pattern, he may reveal himself and so become vulnerable to counterattack. And, it is possible to have both defensive and offensive postures in mind at once. A good player may try to avoid patterns in his own paths of play while searching for exploitable patterns in an opponent's paths.

The idea that players approach their problem in terms of offense and defense suggests a method of inquiry into their behavior. The decision rules I consider have two parts, implemented in two stages in each period. In the first stage at time T, play so far is analyzed for a pattern of serial dependence. If no pattern is detected, no forecast is made. If no pattern is found, I assume that a player obtains what prior research suggests he obtains. In matching pennies that is success with probability 50%[2]. However, if a pattern is detected in the first stage with sufficiently high probability, in the second stage a forecast about time $T+1$ is made, using the model described in section 2. The forecast is that play will extend a pattern, in one of two ways. The pattern may be extended by a choice that continues a representativeness bias. Alternatively, a pattern may be extended by an opponent's best response to a representative

[1]Imagine you have an aid to randomization like a coin, and that you have been meticulously playing your equilibrium mixed strategy in a repeated symmetric matching pennies game, but your coin has now come up heads eight times in a row. You may be strongly tempted to "correct" nature in some way, perhaps by making a non-random choice that brings your sequence closer to your perception of a random sequence.

[2]Note: There is no assumption here that a player can actually implement the serially independent mixed strategy. I only assume that an outside observer was unable to distinguish play from serially independent mixed strategy play.

pattern.

If a player's choices actually are the serially independent Nash mixed strategy, the rules I consider do no worse than random. But if a player's choices do reflect representativeness bias, the decision rules can do significantly and economically better than equilibrium play[3]. One typical result of this two-stage analysis is given in Table I.11, Column 1. In R&B 45 pairs of players in up to 150 rounds created 12,780 overlapping paths of play of length 7. In the first stage, 4,018 paths (combined with older paths in most cases) were assessed as patterned with sufficient probability to move to the second stage. In the second stage 4,018 forecasts were made about the next play; the forecast was right 2,325 times and was wrong 1,693 times, a significant difference from randomness. It is also economically significant: The advantage exceeds, for example, the house advantage in Las Vegas' gambling casinos.

The literature contains several theories of how serially dependent play may arise from local representativeness. One theory of local representativeness uses *balance*, the idea that the next outcome is more "random" if it brings the cumulative frequencies of strategies in the sample closer to the population frequencies. Another theory arises from the idea that, other things equal, change is more unpredictable than persistence. This idea is formalized through a *runs* test. R&B propose a rule based on balance and runs. A related approach is the half-facetiously labelled "law of small numbers". Believers in the law of small numbers act as if they believe that random sequences are actually generated by sampling *without* replacement from a finite set of possible outcomes. Rabin (2002) explores the theoretical implications of the "small numbers" model. In this paper, I use "representative" and "representativeness" to name

[3]Hereafter a rule is *significant* if the difference between forecasts based on the rule and forecasts based on serially independent mixed strategy choices are statistically significant at the 95% level and favor the rule.

Table I.1: A Cycle of Testable Patterns

Pattern	Information Set	Motivation	Relation to Rep.
Rep.	Own Play	Defense	Rep.
BR(Rep.)	Opponent Play	Offense	BR(Rep.)
C-Rep.	Own Play	Defense	BR(BR(Rep.))
BR(C-Rep.)	Opponent Play	Offense	BR(BR(BR(Rep.)))
Rep.	Own Play	Defense	BR(BR(BR(BR(Rep.))))

the boundedly rational behavior at issue. Where the specifics matter, I state them.

There is evidence of representativeness in M&S aggregated across subject pairs, but a full test requires examination of the data pair by pair. The analysis is complicated by the fact that individual subject behaviors are unstable in that subjects may change strategies without necessarily converging over the course of an experimental session. I deal with both problems by allowing for a focus on comparatively brief paths of individual play. I consider four kinds of patterns of play that might reflect representative behavior and forecast continuation of those patterns, see Table I.1. The four kinds of patterned sequences I call representativeness ("Rep"), best response to representativeness ("BR Rep"), counter representativeness ("C-Rep") and best response to counter representativeness ("BR C-Rep"). C-Rep is the best response to the best response to Rep, or play contrary to representativeness. C-Rep is well-defined here since the strategy space is binary. In two patterns, Rep and C-Rep, players are trying to be unexploitable and play defense by conditioning on their own play. In two patterns, BR Rep and BR C-Rep, players are trying to exploit patterns in their opponent's play by conditioning on opponent play and playing offense. Each of the four patterns assumes conditioning on the history of exactly one player, and conditioning on one type of pattern of play; however, linear combinations of the patterns are allowed. A player can have both offense and defense in mind. As shown in Table 1, the best responses to these four kinds of patterns form a closed cycle.

I find that Rep is frequently present (stage one) and forecastable (stage two) in both data sets. BR Rep is infrequent and is not forecastable. C-Rep and BR C-Rep are present and forecastable, but less prominent than Rep.

Section 2 develops the theoretical framework and the econometric specification. Section 3 describes the experimental data of M&S and R&B. Section 4 describes the results of the present analysis. Section 5 is a discussion.

I.B Theory and Testable Implications

The goal of the following formalization is to state the behavioral assumptions carefully and embody them in a form that can be taken to data. The substance of this section starts with a strength function to capture an assumption about a player's mixing probabilities in a repeated fixed-pair constant-sum game. A definition of the path of play follows. A count function is needed to define balance of paths. Balance and runs functions incorporate behavioral assumptions which are combined in the representativeness function. The representativeness function delivers an ordinal score; more representative paths have higher scores. Then there is a formal test of whether a set of overlapping paths are representative and a function to forecast behavior in the presence of representative play. Following that, I develop four similar regressions, one for each of the four kinds of pattern, and then allow for their combination. Finally, there is an example of how this structure behaves. I specialize to the matching pennies game, but the notation foreshadows generalization to mixed strategy games generally.

I.B.1 Definitions, Pattern Detection and Forecasts

Let there be I players in a repeated fixed-player game having a unique mixed strategy Nash equilibrium and let $s_{i,t}$ represent player $i's$ choice in round t from $i's$ strategy space S_i. Let $J_i \in \mathbf{Z}$ be the number of distinct strategies in S_i. In the matching pennies game, $I = J_1 = J_2 = 2$, and $S_1 = S_2 = S = \{0, 1\}$.

Define the *strength* function $\theta_i : S_i \rightarrow (0,1)$ where $\sum_{s_i \in S_i} \theta_i(s_i) = 1$. The strength function is the density function of player i's mixed strategy[4]. In matching pennies, the strength function is simply an ordered pair of non-negative fractions that sum to one.

Player $i's$ l-step *path* i_l is an element of the l-tuple of sets $L = S_i^l$, specifically, the time-ordered sequence of $i's$ most recent strategy choices,

$$i_{l,t} = (s_{i,t-l+1}, s_{i,t-l+2} \ldots s_{i,t}).$$

Hereafter, I suppress the time subscripts when referring to paths to simplify the notation, while retaining reference to the order of choices within a path, and

$$i_l = (s_{i,1}, s_{i,2} \ldots, s_{i,l}).$$

Define the *count* function $c : L \times S_i \rightarrow \mathbf{R}$,

$$c(i_l, s_i) = \sum_{k=1}^{l} I(s_{i,k-l} = s_i) - l\theta_i(s_i),$$

as the function which counts the number of times a strategy s occurs in a path i_l in excess of its strength, where $I(\cdot)$ is the indicator function.

[4]A strictly rational player intends to play the Nash mixed strategy probability with serial independence, but a boundedly rational player may employ either another probability, serial dependence or both. The focus of this paper is on players who play the Nash mixed strategy subject to serial dependence; however, by varying the strength function a researcher could test another mixed strategy with serial dependence. Obvious alternatives for mixed strategy probabilities include strategies that incorporate behavioral assumptions, probability matching for example. Also, in this paper, the strength function does not respond to realized behavior in the game. One could make the strength function endogenous, thereby incorporating Bayesian updating, quasi-Bayesian updating or a learning theory.

The balance function reflects a behavioral assumption: Players view as more balanced, hence "more" random, a path which is closer to that of the population in cumulative (within the path) frequencies of strategies. Define the *balance* function $b : L \times \mathbf{Z} \to \mathbf{R}$ as

$$b(i_l) = J_i \max_{s_i \in S_i} c(i_l, s_i).$$

In matching pennies rational players take $\theta_i(s) = 0.5, \forall s$ and the balance of a path i_l is twice the larger of (i) the frequency of heads minus $l/2$ and (ii) the frequency of tails minus $l/2$. This definition is consistent with R&B.

The runs function reflects another behavioral assumption. Human attempts at random strategies are often characterized by excessive runs. Other things equal, assume paths with more runs are perceived as more "random". Define the *runs* function $r : L \to \{1, 2, \ldots l\}$ as

$$r(i_l) = 1 + \sum_{k=1}^{l-1} I(s_{i,k-l} \neq s_{i,k-l+1}).$$

This is a formalization of the standard definition. The idea that paths with more runs are perceived as more random is not as well motivated as the notion of balance. The notion of the number of runs as a measure of perceived randomness is clearly not persuasive at the extreme of maximal runs: Simple alternation is not perceived to be particularly random. A useful extension of this paper would be a better runs function.

Both the balance and the runs tests are ordinal; they provide no quantitative information with which to measure the magnitude of the difference between paths. The definition of representativeness will also be ordinal. Representativeness is embodied here in a lexicographic weak ordering of paths by balance, breaking ties in balance with runs. Define the *representativeness function* $br : L \to \mathbf{R}$ as

$$br(i_l) = lb(i_l) + r(i_l),$$

and call $br(i_l)$ the representativeness score (or just *score*) of path i_l. Relatively representative paths have relatively high scores. The representative ordering could be defined with other notions of representativeness; this definition is drawn from R&B.

The next step is to develop a formal hypothesis test of whether a set of paths is representative. A player generates $t - l + 1$ overlapping paths up to time t. Each path is a draw from a set of 2^l possible paths. Given θ_i, one can work out the population probability distribution of these path frequencies conditional on independent play and transform the distribution into an ordering over path frequencies. Treat this ordering of path frequencies as the null hypothesis. For the alternative hypothesis consider the representative ordering of path frequencies. One can then test whether path frequency distributions as highly or more highly characterized by representativeness are likely to be a draw from the null. If the null is rejected, *representativeness* is confirmed.

In general it is possible that the representative ordering could be so close to the null ordering that the test has little power; however, the ordering of path frequencies for the equilibrium mixed strategy takes a particularly convenient form in matching pennies. Since every play is equally likely, the distribution of path frequencies is uniform in large samples[5]. The null hypothesis is zero correlation. So, a set of empirical matching pennies paths is representative if the rank correlation between its empirical path frequencies and the representative ordering of path frequencies is significantly greater than zero.

Next consider estimators of correlation between an empirical set of paths, the representative ordering and the null. Since scores are ordinal, an estimator of rank correlation in needed. There are two standard rank correla-

[5]In finite samples, there will be some sampling variation in path frequencies, but no *ex ante* ordering.

tion estimators, Kendall's $\hat{\tau}$ and Spearman's $\hat{\rho}$. I use both, as well as versions of each that adjust for ties in rank, τ and ρ. A researcher's choice among these four measures makes little difference. Formulas for the tie adjustments are given in Appendix A. Critical values for overlapping paths are obtained using a parametric bootstrap technique. That also is discussed in Appendix A.

Having developed a test for representative play in this framework, I now turn to forecasting. Suppose, in choosing her next play, player i conditions on the history of exactly one player j. If $j = i$ then player i conditions on her own history, and plays *defense*; if $j \neq i$ then player i conditions on the history of another player and plays *offense.* In either case, a player uses history to choose from feasible alternatives. Let j_l be the path consisting of the last l choices. Define $j_l|s_i$ to be the concatenation of j_l and the action s_i, which is a path of length $l + 1$. Then

$$\hat{br}(j_l|s_i) = arg\ max_{s_i \in S_i} br(j_l|s_i)$$

gives the current action s_i which maximizes the representativeness score given the last l choices. If $j = i, \hat{br}(j_l|s_i)$ is the *representative forecast,* the most representative conditional choice. If $j \neq i$, and i wins by matching with j, $\hat{br}(j_l|s_i)$ is the best response to representativeness in j.

I.B.2 Forecasting Models

Having a test for a representative pattern and a tool for forecasting representative play, I now formalize models to take to data. I use the first stage to find play with a pattern. Conditional on finding a given pattern with sufficiently high probability, the second stage forecasts by selecting play that continues (defense) or best responds (offense) to the historical pattern. I start from a general functional form for forecasting and specialize to the testable forms in several steps.

Let there be I repeated fixed-pair matching pennies games, each player playing T rounds. Let $s_{i,t}$ be the decision of the row player (Row) in game i at time t, where $s_{i,t} \in \{0,1\}$. The column player (Column) chooses $s_{j,t} \in \{0,1\}$.

A general functional form for forecasting of $s_{i,t}$ is

$$\begin{aligned} s_{i,t} &= f(s_{i,t-1-k}, s_{j,t-1-k}, X_i'\gamma, \epsilon_{i,t}), \\ & \quad i = 1, \ldots I, t = 1, \ldots T, 0 < k < t \end{aligned}$$

The sets of variables $\{s_{i,t-1-k}\}$ and $\{s_{j,t-1-k}\}$ are i's information set. Row player i may recall her prior play $\{s_{i,t-1-k}\}$, and her opponent's prior play, $\{s_{j,t-1-k}\}$. X_i is fixed effects; γ is its vector of coefficients. $\epsilon_{i,t}$ is an error term.

I assume Row will condition on exactly one player's history for any one specification. The previous form simplifies to:

$$s_{i,t} = f(s_{j,t-1-k}, X_i'\gamma, \epsilon_{i,t}), i = 1, \ldots I, t = 1 \ldots T, 0 < k < T.$$

If $j = i$ then the player is defensive, conditioning on own play; if $j \neq i$ then the player is offensive, conditioning on the other player's play.

A subset of the prior paths—a window—is used in the first stage to identify the presence of a pattern in the periods prior to period t. Early in a game, some periods t are rejected at the first stage because there is not enough history; in later periods t there is a lot of history and the oldest paths are not used. Specifically, a period t can pass the first stage only if there are at least $\underline{w}$ prior overlapping paths to consider. At most, the $\overline{w}$ most recent overlapping paths are used. If at time t the number of historical paths is between $\underline{w}$ and $\overline{w}$ paths, all paths are used.

Note that the window may extend much further into the past than the player's assumed memory of l specific past plays. A player need not remember the details of play more than l periods ago in order to implement a pattern like representativeness over a time span longer than the player's memory. This is fortunate because the researcher may need more than l periods to identify a pattern with confidence.

Whether a pattern is sufficiently probable is assessed using rank correlation estimator $r(\cdot)$, and threshold probability x. The argument to $r(\cdot)$ is (i) a set of path frequencies aggregated over the specified window, (ii) the null hypothesis, which for symmetric games like matching pennies is a uniform distribution of path frequencies, and (iii) an alternative representativeness hypothesis about expected frequencies. In the notation I have compressed this to refer only to which player's paths are rank correlated. The rank correlation of the relevant subset of player j's paths is referred to as $r(j)$. There is a one-to-one function between critical values for $r(\cdot)$ and p-values, which function is also suppressed in the notation.

Formally, the first stage is an indictor function. The function is one if at least $\underline{w}$ prior overlapping paths exist at time t and if representativeness in the $\min(\overline{w}, t-l)$ preceding paths of length l of player j using rank correlation measure $r(\cdot)$ has probability in excess of the correlation threshold x. An expression for the indicator is $I(r(j) > x)$. Aside from robustness checks, $\underline{w} = 1, \overline{w} = 50, x = 0.8$, and $r(\cdot) = \rho(\cdot)$. Let $\hat{br}(j_l|s_i)$ be the forecast of i's play. The specification becomes:

$$\begin{aligned} s_{i,t} &= f(I(r(j) > x)\hat{br}(j_l|s_i)\beta, X_i'\gamma, \epsilon_{i,t}), \\ & \quad i = 1, \ldots I, t = \underline{w} + l, \ldots T. \end{aligned}$$

Since $s_{i,t}$ is a limited dependent variable, I use a logit functional form.

$$s_{i,t} = f_L(I(r(j) > x)\hat{br}(j_l|s_i)\beta + X_i'\gamma + \epsilon_{i,t}), \tag{I.1}$$

$$i = 1, \dots I, t = \underline{w} + l, \dots T,$$

where

$$f_L(z) = \frac{e^z}{1 + e^z}.$$

The parameter of interest is β. If $j = i$ we test Rep and the coefficient is labelled β_1, if $j \neq i$ we test whether i has a history of best responding to representativeness, BR-Rep in j and the coefficient is labelled β_2.

Define C-Rep as negative rank correlation. Because counter-representative play lacks balance and runs, it tends to be streaky relative to serially independent play. Forecast C-Rep players by guessing that their choices will continue to be counter representative. Hence the C-Rep specification:

$$\begin{aligned} s_{i,t} &= f_L(I(r(j) < -x)(1 - \hat{br}(j_l|s_i))\beta + X_i'\gamma + \epsilon_{i,t}), \\ & i = 1, \dots I, t = \underline{w} + l, \dots T, \end{aligned} \tag{I.2}$$

If $j = i$ we test C-Rep with β_3; if $j \neq i$ we test best response by i to counter-representativeness in j BR C-Rep with β_4.

One can linearly combine any permutation of these models and get a new model. The most general is the portmanteau model:

$$\begin{aligned} s_{i,t} &= f_L\,[\, I(r(i) > x)\hat{br}(j_l|s_i)\beta_1 \\ &+ I(r(j) > x)\hat{br}(j_l|s_i)\beta_2 \\ &+ I(r(i) < -x)(1 - \hat{br}(j_l|s_i))\beta_3 \\ &+ I(r(j) < -x)(1 - \hat{br}(j_l|s_i))\beta_4 \\ &+ X_i'\gamma + \epsilon_{i,t}\,], i = 1, \dots I, t = \underline{w} + l, \dots T. \end{aligned}$$

Here I assume $j \neq i$ and so β_1 is the coefficient on Rep forecasts, β_2 is BR-Rep, β_3 is C-Rep and β_4 is BR C-Rep.

Forecasts from components of the portmanteau model might conflict; however, most conflicts are avoided by construction. Except for robustness

Table I.2: 3-step Paths

Path i_3	Balance $b(i_3)$	Runs $r(i_3)$	Score $br(i_3)$
(0,0,0)	0	1	1
(0,0,1)	1	2	5
(0,1,0)	1	3	6
(0,1,1)	1	2	5
(1,0,0)	1	2	5
(1,0,1)	1	3	6
(1,1,0)	1	2	5
(1,1,1)	0	1	1

checks, the correlation threshold x is always set to a positive value. If the correlation threshold x is positive, then at most one of the defensive models (Rep and C-Rep) can be found to be likely enough to merit a second-stage forecast at a particular t; similarly at most one of the offensive models (BR Rep and BR C-Rep) will merit forecasting. It remains possible for offensive and defensive strategies to produce contradictory forecasts. This theoretical possibility is important for verisimilitude; however, (see Table I.11, Panel B, Column 1 and related text) it is not a problem in the data. The portmanteau model is consistent with a variety of behaviors, including contradictory ones. The model is robust to variation among players and instability or convergence of play, but does not characterize variation, instability or convergence.

I.B.3 Matching Pennies Example

An example of representativeness in matching pennies follows. Suppose player 1 plays heads with probability 50%; let $\theta_1(H) = 1/2$.

Consider matching pennies paths of length $l = 3$. Table I.2 shows the 2^3 possible paths along with their balance, runs and score. The path $i_3 = (0,0,0)$ has score 1 and is relatively unrepresentative. On the other hand,

Table I.3: Four-step Paths in the Thought Experiment

Path i_4	Balance $b(i_4)$	Runs $r(i_4)$	Score $br(i_4)$
(0,0,0,1)	1	2	6
(0,0,1,1)	2	2	10
(0,1,0,1)	2	4	12
(0,1,1,0)	2	3	11
(1,0,0,1)	2	3	11
(1,0,1,0)	2	4	12
(1,1,0,0)	2	2	10
(1,1,1,0)	1	2	6

path $i_3 = (1, 0, 1)$ has score 6 and is highly representative. The paths fall into three equivalence classes of scores. The representative ordering of scores for $l = 3$ is

$$w_6 Pr[br(i_3) = 6] > w_5 Pr[br(i_3) = 5] > w_1 Pr[br(i_3) = 1]$$

where w_k is a weight[6]. Here $w_6 = w_1 = 1/2, w_5 = 1/4$. The weights can be obtained by tabulating frequencies of scores reported in the last column of Table I.2.

Consider now a thought experiment in which a player is truly serially independent for three rounds, but is always representative in the fourth and last round of a series of non-overlapping paths. I will show how the tools just developed detect representative play in the player's path history.

[6]The weights w_k are a necessary nuisance that arises from ties in scores for different paths. Each of the 2^l possible paths has one of not more than l^2 distinct scores. There is a many to one mapping from distinct paths to distinct scores. Some scores may be more frequent because a larger number of distinct paths are assigned that score. Differences in the number of distinct paths having the same score must be taken into account. If k is the score of an equivalence class of paths, let w_k be the inverse of the number of distinct paths in the class. If play is representative, w_k times the number of empirical paths with score k will have the representative ordering. Formally, for representative play, if x and y are two valid numerical scores and $x > y$ then

$$w_x Pr[br(i_l) = x] > w_y Pr[br(i_l) = y],$$

where $Pr[\cdot]$ refers to empirical frequencies.

Table I.4: Four-step Representative Equivalence Classes

Score $br(i_4) = k$	Weight w_k
1	1/2
6	1/4
7	1/4
10	1/2
11	1/2
12	1/2

Given three random plays, the distribution of 3-step paths will be uniform and each of the eight paths (see Table I.2) is equally likely. If the player now chooses representative play, the result is the set of eight 4-step paths given in Table I.3. Table I.4 sorts these eight paths plus all the other possible 4-step paths into equivalence classes of scores and gives weights w_k. The representative ordering of 4-step scores is:

$$\begin{aligned} w_{12}Pr[br(i_4) = 12] &> w_{11}Pr[br(i_4) = 11] \\ &> w_{10}Pr[br(i_4) = 10] \\ &> w_7 Pr[br(i_4) = 7] \\ &> w_6 Pr[br(i_4) = 6] \\ &> w_1 Pr[br(i_4) = 1] \end{aligned}$$

Rank correlation of the 3-step paths is zero by construction, but when the next play is representative, the correlation becomes notable. By inspection, 10 of the 15 inequalities in the representative ordering hold for the data in Table I.3; there are 4 ties and 1 inequality fails. As illustration of one of these inequalities consider scores 7 and 12. In Table I.3 no paths have score 7, so the weight-adjusted frequency of scores of 7 is zero. Two paths have score 12,

Table I.5: Rank Correlations in the Thought Experiment

Rank Correlation $r(\cdot)$	Value
ρ	0.61
$\hat{\rho}$	0.66
τ	1.20[7]
$\hat{\tau}$	0.87

with weight 1/2, so the weight adjusted frequency of scores of 12 is 1. Since

$$w_{12}[Pr(br(i_4)) = 12] = \left(\frac{1}{2}\right)(2) > w_7[Pr(br(i_4)) = 7] = \left(\frac{1}{4}\right)(0)$$

the representative ordering holds for this pair of scores in this hypothetical data.

Table I.5 reports rank correlations for the thought experiment. Since the paths do not overlap, I cannot assess p-values for these correlations using the technique I use in the remainder of the paper; however, all the correlation measures are highly positive.

I.C Data

M&S and R&B report experiments and the data are from those experiments[8]. Both experiments were repeated fixed-pair symmetric constant-sum games like the school yard version of matching pennies, Figure I.1.

The M&S and R&B experiments are well-suited to the study of representativeness. Matching pennies is a game of pure competition. There is no element of coordination and so no complication from attempts to coordinate on patterns, as in Sonsino (1997). In both data sets, players typically play

[7]The importance of ties is apparent. The tie-adjustment formulas do not confine reported statistics to $[-1, 1]$, as illustrated here for Kendall's τ with tie adjustment. Hereafter, except as noted, I use p-values associated with correlations rather than the correlations themselves.

[8]I thank both Barry Sopher and David Budescu for their data.

pure strategies with frequencies that closely approximate over time the equilibrium mixed strategy probabilities. Matching pennies is the simplest game in which representativeness plays an important role[9].

M&S conduced their experiment as a positive sum game. Subjects were economics students at the University of Delhi. Ten pairs of players played 40 rounds of two treatments. I use only Treatment 2[10]. R&B (Treatment D, "Dyad") was a psychology experiment. Forty-five pairs of undergraduates from the University of Haifa played 150 rounds of matching pennies[11].

In both experiments, a repeated fixed-pair time series of Row and Column strategy choices comes from the Row perspective. Column is isomorphic to Row, and by transforming the data—by reversing Column's strategies (assigning 0 to 1 and 1 to 0) and assigning the Row label to Column and vice versa—a second repeated fixed-pair time series is available. Each subject appears in exactly one series as Row and in exactly one other series as Column. The connection between the two series is ignored hereafter.

Both papers report descriptive statistics which I do not repeat. I report one new tabulation of the R&B data to show some structure reflected in relative frequencies of paths of play. I consider the set of all non-overlapping paths in the R&B data. For selected sets of paths of an even length, I report the number of such paths, the number that are balanced (equal number of heads and tails), the number of streaks (all heads or all tails) and the rest. Along with these empirical frequencies, I report the ratio between the empirical and expected number of paths for each type of path. The expectation is based on

[9]There may be many games in which players commonly fail serial independence and fail to match their play to the optimal mixing probability. This is an area for future work.

[10]Treatment 2 was the standard matching pennies game; in Treatment 1 subjects were not told the payoff matrix, opponent's choices or opponent's realized payoffs.

[11]Subjects received an initial endowment of 20 New Israeli Shekels. Some lost their endowment and the game terminated. Consequently, the data is an unbalanced panel. R&B had two other treatments that had no strategic interaction.

Table I.6: R&B Data: Balanced, Streaky and Other Play

Path length	N	Balanced	Balanced/ E[balanced]	Streaks	Streaks/ E[streaks]	Rest	Rest/ E[rest]
2	6184	3251	1.05	2933	0.94	—	—
4	3088	1348	1.16	316	0.81	1414	0.92
6	2010	794	1.26	40	0.64	1176	0.89
8	1504	542	1.31	7	0.60	955	0.88
12	1000	300	1.32	1	2	699	0.90
20	582	159	1.55	0	—	423	0.88

	0	1
0	0 1	1 0
1	1 0	0 1

Figure I.1: Matching Pennies Normal Form Game

the null hypothesis that play is serially independent 50% heads. For example, there are 1504 paths of length 8, 542 of which are balanced and 7 are streaks, 1.31 times and 0.60 times (respectively) as frequent than one would expect in random play.

I.D Results

The first test is for representativeness in M&S aggregated across subject pairs. Rank correlations between empirical and representative path frequencies from the M&S data are reported in Table I.7. Two different measures of correlation, Kendall's τ and Spearman's ρ (both with tie adjustment) ap-

Table I.7: Representativeness in M&S Data

Path Lengths l	Kendall's τ	Spearman's ρ
4	0.64*	0.73*
5	0.47*	0.60*
6	0.38*	0.49*
7		0.39*
8		0.32**
9		0.24**

10%, ** 5%

pear, with path lengths from 4 to 9. All of the reported statistics are significant at least at the 10% level. For the case of binary strategies analyzed in this paper, path lengths shorter than $l = 4$ and longer than $l = 9$ are difficult. Paths shorter than $l = 4$ are difficult because there are only 8 paths of length 3 (*see* Table I.2). For paths of length $l = 9$, there is plenty of theoretical variation—$2^9 = 512$ possible paths—but the frequencies may be poorly estimated for lack of data. In M&S expected path frequency for path length $l = 9$ is about 1.2.

If paths are representative at length $l = z$ they may be representative at $l = z + 1$ since the paths differ only in the $z + 1$'th element. So the tests reported in Table I.7 are not independent, but neither are they redundant; they are mutually supportive in that they all have the right sign and similar levels of statistical significance. This is evidence for the representative play in the M&S data as a whole. I now turn to forecasting models.

Table I.8, Column 1 gives results for the first Rep specification. Path length is $l = 5$; a single constant is estimated instead of pair fixed effects. With the correlation statistic set at 80% probability, 115 out of 680 observations, involving 8 of the 20 players, passed the first stage. In the absence of a consensus measure of goodness-of-fit for logit models (*see,* Greene (2000),

Table I.8: M&S Data, $l = 5$

Model	Rep.	Rep.	Rep.	Port.	Rep, Lags
Path Length l	5	5	5	5	5
Fixed Effects	No	Yes	No	No	No
Function	Logit	Logit	OLS	Logit	Logit
Forecasts	115	115	115	271	
Pairs used	8	8	8	18	
Percent correct	66.1	66.1	66.1	59.8	
Net wins	37	37	37	53	
$\hat{\beta}_1$ z-score	3.38	3.57		3.42	3.25
$\hat{\beta}_1$ t-stat			3.77		
Probability $\beta_1 = 0$	0.0004	0.0002	0.00008	0.0003	0.0005
$\hat{\beta}_2$ z-score				0.46	
$\hat{\beta}_3$ z-score				0.94	
$\hat{\beta}_4$ z-score				1.64	
$C_{i,t-1}$ z-score					1.66
$C_{i,t-2}$ z-score					-1.52
$C_{i,t-3}$ z-score					0.05
$C_{i,t-4}$ z-score					0.11

p. 831), I use three measures: (1) percent correct, conditional on a forecast (sometimes called "efficiency") (2) net wins and (3) $\hat{\beta}$ z-score. Percent correct has a transparent intuition: 50% is worthless, 100% is perfect. The representativeness forecast was right 66.1% of time time, 37 times more than it was wrong. The z-score on $\hat{\beta}_1$ is 3.38, p-value 0.0004. In this regression $\hat{\beta}_1 = 1.33$. This coefficient is not reported hereafter since it is its sign and statistical significance that matters. Rep is both a statistically and substantively significant forecaster of play in M&S.

Table I.8, Column 2 replaces the constant with pair fixed effects. An F test for the fixed effects as a whole was not significant; for 2 players out of 20 fixed effects were significant at the 90% level, consistent with insignificance. A constant is used instead of fixed effects hereafter.

Table I.9: M&S Path Length l Variations

Model	Rep.	Rep.	Rep.	Rep.	Rep.	Rep.	Rep.
Path Length l	3	4	5	6	7	8	9
Forecasts	19	129	115	143	143	137	116
Pairs used	1	9	8	8	8	9	8
Percent correct	78.9	61.2	66.1	60.1	63.6	59.1	60.3
Net wins	11	29	37	29	39	25	24
$\hat{\beta}_1$ z-score	2.33	2.52	3.38	2.41	3.22	2.12	2.22
Probability	0.01	0.006	0.0003	0.008	0.00006	0.017	0.013

Now I vary parameter values to explore robustness. First, vary the path length, adjusting the rank correlation estimator to maintain 80% probability of a pattern. This is summarized in Table I.9. The results are statistically significant for all paths lengths from $l = 3$ to $l = 9$. Net successes range from 11 to 39. Success rates vary from 59.1% to 78.9%. The first and second best on all three of the ranking measures—percent correct, net wins and z-score—are odd-length paths. Odd path lengths cannot have ties in balance, but even path lengths may require resort to runs as a tie-breaker. Odd path lengths doing better is consistent with the runs test being a poor tie-breaker.

Consider varying the rank correlation probability. Net successes vary from 14 for $x = -1$ (no first stage) to 43 ($x = 0.20$[12]) and then falls to 16 ($x = 0.80$[13]). Percent correct is only 51% with no first stage, but increases smoothly to 77% for $x = 0.80$. The z-score varies from 0.54 (no first stage) to 3.87 ($x = 0.50$) and then falls to 2.76 ($x = 0.80$). One can obtain non-significant results for $x < |0.20|$. This is evidence that some play is unforecastable and a first stage is necessary.

Now consider varying the rank correlation statistic: There are four to choose from and they are all significant. Spearman with no tie adjustment

[12] Corresponds approximately to $Pr > 60\%$

[13] Corresponds approximately to $Pr > 98\%$

$\hat{\rho}$ is worst, with z-score 2.90. Percent correct ranges from 60-75%; net wins 21-41. No one measure dominates.

Similar robustness tests were conducted by changing the conditioning window, parameterized by $\underline{w}$ and $\overline{w}$. Increasing $\underline{w}$ does not matter unless it begins to decrease the amount of available data significantly. In the baseline specification, Table I.8, Column 1, changing $\underline{w} = 1$ to $\underline{w} = 15$ reduces the z-score for the representativeness variable from 3.38 to 3.20. Decreasing $\overline{w} = 50$ to $\overline{w} = 9$ in the baseline specification reduced the z-score from 3.38 to 3.34. The algorithm requires about $l + 9$ periods of play to make optimal (as a function of the number of periods analyzed) forecasts. For still smaller windows, the results begin to deteriorate. To find serial dependence this approach requires a number of periods to detect the dependence. If players change models too frequently, the model will be unable to detect their patterns.

In sum, the favorable results for Rep are robust to parameter variations. Here is a list of the parameters that could have been manipulated in search of a result for baseline representativeness specification, Table I.8, Column 1: path length l, trigger level x, correlation statistic $r(\cdot)$, minimum window $\underline{w}$, maximum window $\overline{w}$, fixed effects (yes/no), logit v. OLS, and permutations of linear combinations this model with other models (reported later). Without exception, the representativeness results are robust to modest changes. One might also manipulate the data directly, omitting troublesome players, games or time periods. Here all the data were used. Or, one might use a different path length l for the first stage than for the second stage forecast. I did not.

As another check on hidden dependencies due to overlapping paths, for example, or data mining, I simulated a random data set using the bootstrap procedure described in Appendix A (but only one replication) and ran it through paces similar to those for the M&S data. I tried 10 regressions with different path lengths, triggers, statistical measures, and windows, all for the Rep model. The regression with the best $\hat{\beta}$ had p-value 11%, though the sign was wrong.

Having found Rep in M&S, consider the three other models. The portmanteau model is characteristic, Table I.8, Column 4. BR Rep, β_2, and C-Rep, β_3, are not statistically significant either here or separately. BR C-Rep, β_4, is barely statistically significant both in Column 4 and when run alone at the 95% level, but the result is not notably robust to varying parameters (not reported).

While there is no evidence of BR Rep β_2 and weak evidence of BR C-Rep β_4 in M&S, there is slightly stronger evidence of players responding to their opponents in another way. Table I.8, Column 5 shows Rep with lags of opponent's play. (No results are reported for success measures because I have no theory for how to assess success for Rep with lags.) Opponent's lags one and two are not significant alone (M&S have this result), but opponent's lags one and two are statistically significant, with opposite signs, when combined with the Rep model. The result is robust to dropping opponent's insignificant lags 3 and 4, as well as parameter variations.

The M&S data was completely analyzed before any review of the R&B data. The specification first developed in M&S was applied to the R&B data without modification. The critical values for $r(\cdot)$ are specific to the number of games and their length in M&S. Nevertheless, for consistency, I retained the same critical values.

Results for Rep in R&B data are given in Table I.10, Column 1. 3,558 forecasts were made with 568 net wins (58%) and z-score 9.46, probability 0.0000000. Column 2, BR Rep, is not significant and has the wrong sign.

Table I.10: R&B Data, Path Length $l = 5$

Model	Rep.	BR(Rep.)	C-Rep.	BR(C-Rep.)
Path Length l	5	5	5	5
Forecasts	3558	3630	1158	1129
Net wins	568	-84	54	85
Percent correct	58.0	48.8	52.3	53.8
$\hat{\beta}$ z-score	9.46	-1.44	1.56	2.46
Probability	0.0000000	0.07	0.058	0.007

Column 3, C-Rep, just misses statistical significance. Column 4, BR C-Rep is statistically significant, but not nearly as salient as Rep.

The R&B game is almost four times longer than the M&S game, for those who went the distance. Perhaps in a longer game, longer paths matter. Longer paths also means more possible paths, and allowing the path space to ramify is more promising in a larger data set. Path frequencies may be more precisely estimated in a larger data set. Table I.11, Panel A is the same as Table I.10, but with path length changed from $l = 5$ to $l = 7$. Rep is as prominent as before and BR Rep retains the wrong sign and remains insignificant; however, the borderline results for C-Rep improve to strong statistical significance. BR C-Rep remains significant.

The three significant models, Rep, C-Rep and BR C-Rep are combined in Table I.11, Panel B, Column 1. When forecast separately, 5212 forecasts are made in defensive strategies (Panel A, Columns 1 and 3) and 1138 in offensive strategies (Panel A, Column 4). When offensive and defensive strategies are combined, the number of forecasts is less than the sum of the parts, because there are cases in which there is both an offensive and a defensive forecast. When they agree, the forecast is used. When they disagree, no forecast is made and that observation is treated as not passing the first stage. With this rule for combining potentially conflicting strategies, the number of net wins increases. By the net wins measure, offense and defense together outperform either alone. The win rate is virtually identical to the weighted average win rate of the parts that make up the combined model. In sum, combining offense and defense increases the number of forecasts versus either offense or defense alone, and does not greatly affect the quality of the forecasts.

Table I.11, Panel B, Column 2 adds the first lag of opponent's play to Column 1. When a forecast is made for all three of the component models, the conflict is resolved by using the forecast made by at least two of the three models. The first lag is significant. The first lag does not affect the defensive models (Rep and C-Rep). The first lag reduces but does not eliminate the significance of offense (BR C-Rep). Longer lags were insignificant.

Table I.12 repeats the exercise for path length $l = 9$. Rep, Column 1, is as strong as before, but makes about 20% more forecasts and gets them right with almost the same probability as for $l = 5$. A standard subroutine in Gauss, QNewton Version 5.0.14 could not solve BR Rep because a matrix was complex—perhaps consistent with its being insignificant. C-Rep, Table I.12, Column 2, is now also extremely significant. BR C-Rep, Table I.12, Column 3, is slightly weaker than with $l = 7$, though still significant.

I.E Discussion

At times players choose forecastable strategies. Overall, the strongest patterns were for path length $l = 7$ in the R&B data, Table I.11 and length $l = 5$ in the M&S data, Table I.8, Column 1.

A player trying to play unexploitable defense may be representative instead. Rep was prominent in both of the data sets, one conducted by economists with Indian subjects, the other by psychologists with Israeli subjects. In many R&B specifications, when it guessed, Rep was right about four

Table I.11: R&B Data, Path Length $l = 7$

Panel A

Model	Rep.	BR(Rep)	C-Rep.	BR(C-Rep)
Path Length l	7	7	7	7
Forecasts	4018	4070	1194	1138
Percent correct	57.9	49.3	55.9	53.0
Net wins	632	-60	142	68
$\hat{\beta}_1$ z-score	9.92	-0.95	3.98	2.01
Pr $(\beta_1 = 0)$	0.0000000	0.17	0.00003	0.02

Panel B

Model	Rep.,C-Rep. BR(C-Rep.)	plus lag
Path Length l	7	7
Forecasts	5576	
Percent correct	57.2	
Net wins	806	
$\hat{\beta}_1$ z-score	9.97	9.96
Pr $(\beta_1 = 0)$	0.0000000	
$\hat{\beta}_3$ z-score	4.00	3.97
$\hat{\beta}_4$ z-score	2.21	1.83
$\hat{C}_{i,t-1}$ z-score		-3.11

Table I.12: R&B Data, Path Length $l = 9$

Model	Rep.	C-Rep.	BR(C-Rep.)
Path Length l	9	9	9
Forecasts	4210	1147	1067
Percent correct	57.4	59.8	53.1
Net wins	622	225	67
$\hat{\beta}$ z-score	9.54	6.48	2.02
Probability	0.0000000	0.0000000	0.02

times in seven on average, over thousands of rounds.

A player may play offense, seeing her opponent's representativeness and best responding to that. However, BR Rep was not found, perhaps because a typical player's model of random behavior is close to Rep and so the player finds representativeness difficult to distinguish from random behavior.

C-Rep may represent faulty defensive randomization. Alternatively, a player may anticipate BR Rep, whether or not it is actually present, with C-Rep. Whatever the source, C-Rep was significant in R&B data.

Fourth, a player may adopt BR C-Rep. BR C-Rep was statistically significant in R&B and marginal in M&S, indeed BR C-Rep was sometimes more prominent than the C-Rep to which it is the best response. The explanation may be subtle: If a player adopts a counter representative pattern or a random sequence that looks counter representative due to sampling variation, her opponent may be extremely sensitive to it, since there is more difference between the opponent's model of unexploitable play and counter representative play than there is between truly unexploitable play and counter representative play. Further, having perceived counter representative defense, a rational player's best response is deterministic. When the player responds with high probability to a subtle, or non-existent, signal, it is easier to detect the signal plus response than the signal plus the next signal because the signal plus response has a stronger pattern.

Any of these strategies may be thought of as sophisticated—all can be interpreted as a response to an opponent—but Rep and C-Rep could simply be faulty randomization.

The models contain parameters that determine where and how to look for a pattern, but none that quantitatively calibrate a player's behavior. The only parameter estimates relate to whether an entire pre-determined model is statistically significant. For example, the coefficient β_4 for BR C-Rep measures whether a forecast that a player will best respond is significant. Since the specification of player behavior is non-parametric, some kinds of errors in estimation of parameters do not arise. Specifically, the difficulties in inference due to parameters estimated by autoregressions in panel data described by Wilcox (2003) are absent.

The framework is flexible but also harbors structure. A model that used all the information in the information set would use all the history of both players as conditioning information, perhaps with dummy variables for each possible history. Even for the simple matching pennies game that requires 4^n dummy variables, where n is the length of history. I cut this down in several steps: (1) I assume memory is limited so only the past l plays are recalled, where $l < 10$. (2) I suppose that players condition on exactly one player's history in one consistent way in recognizing patterns, though players may test more than one pattern. (3) I suppose that paths are assessed using balance then runs, reducing 2^l discrete paths to not more than l^2 ordered equivalence classes of paths which can be summarized in a single ordinal. These steps respond to the curse of dimensionality.

The balance and the runs tests are not built on linear combinations of prior decisions; they are non-linear. The balance and runs tests are also not linear combinations of functions of prior decisions; they are not additively separable. For example, suppose the relevant history is 11011. The balanced choice conditional on this history is 0, generating 110110. If the most recent choice had been 0, so the history was 11010, the balanced choice would be the same as before, generating 110100. Given the history, the most recent choice has no affect on the balanced continuation. This is one consequence of non-linearity. In linear and additively separable models there are no such interaction effects. Fictitious play models, for example, are built on simple averages of prior play. This may explain why prior work has not found the prominent patterns found here: Linear models may find nothing when a non-linear model is the correct specification.

If the model is intended as one of how boundedly rational people actually make decisions, the model needs to be simple. The balance test and the runs are intuitively transparent while suggesting specifications that conventional regressions may miss.

I.E.1 Comparison to Learning Theory

The most familiar alternatives to Nash equilibrium behavior in mixed strategy games are learning theories. The core idea of learning theory is that players experiment with different mixed strategies, observe the results and then respond to the perceived results of their strategy. Two major modeling perspectives are reinforcement learning and beliefs-based learning. Reinforcement learning is a stimulus/response model, Roth and Erev(1998). Beliefs-based learning entails responses to a model of opponent behavior. Camerer & Ho (1999) offer a hybrid they call experience weighted attraction (EWA). The M&S treatments were designed to distinguish reinforcement from belief-based strategies experimentally. Camerer and Ho (1999) used M&S data to test EWA and Salmon (2001) used simulations to assess the power of the M&S setup to distinguish between different kinds of learning behavior. Perhaps because the equilibrium mixture is so obvious, learning theory researchers have found little evidence of learning behavior in matching pennies games; however, they have not tested representative nonlinear additively non-separable specifications, or learning about such specifications.

O'Neill (1987) conducted a more complicated mixed strategy experiment. There were several alternatives, and one had distinct payoff implications. O'Neill found little evidence of play inconsistent with minimax. Brown & Rosenthal (1990) re-analyzed O'Neill's data and found play inconsistent with randomization, but failed to offer a structural alternative to minimax. Crawford & Iriberri (2005) offer a model of the first round of the O'Neill game motivated by framing and focal point issues. Both Brown & Rosenthal and O'Neill (1991) recognize that individuals have difficulty producing and recognizing serially independent sequences and that any rational agent approach should take that limitation into account.

Fudenberg & Levine (1999) allow for arbitrary temporal patterns and fictitious play in arriving at a mixture, but do not consider the additional structure supplied by representative patterns. Rabin (2002) explicitly theorizes about representative serial dependence. This paper pursues those leads empirically.

It is hard to interpret the behavior identified here as learning because the identification strategy does not require evolution of behavior or convergence over time. Players may learn that, say, BR C-Rep is a good model and so adopt it with increasing frequency when an opponent is perceived to be C-Rep. Players may adopt this approach from the beginning of the game, or develop it over time. The econometric specification is robust to either possibility, but does not distinguish between them.

I.E.2 Contribution of this Paper

Economists who research alternatives to Nash equilibrium behavior in mixed strategy games may have subtle models, but they usually end up with straightforward tests. Most often they are concerned with whether the mixture probabilities reflect the equilibrium or best responses, or evolution toward either equilibrium or best responses. For example, if a player's Nash strategy is 80% heads, does the player generate (or evolve toward generation of) a cumulative frequency of 80% heads?

When they consider sequences of play, economists most often explore a linear model by using dummy variables for each lag. A few researchers consider a non-linear test which bears closer resemblance to the work in this paper. I am aware of two types of tests which might relate to representativeness: a runs test and a kind of autocorrelation test. M&S report runs tests and have three rejections at the 5 % level out of 20 tests, when one is expected. This is weak support for violation of the runs test. Walker & Wooders (2001) find stronger evidence for a runs test violation in their tennis data, but Palacios-Huerta (2003) finds none in his soccer data. Other researchers have looked for excessive alternation in strategies or responses to streaks, usually ascribed to the gambler's fallacy. Perhaps the most interesting of these is Croson & Sundali (2005), who examine streaks of varying length.

No study of which I am aware uses the distinction between defense and offense as a methodological tool for the analysis of repeated mixed strategy games. So far as I am aware, no economist has explored balance in a data set, though Rabin (2002) considers it from a theoretical perspective.

Representativeness in general and balance in particular is familiar to psychologists, and I have borrowed heavily from R&B at the conceptual level. R&B report a correlation statistic for an ordering close to the one I use, but propose no way of assessing statistical significance. The parametric bootstrap in Appendix A is the step that facilitates a test for statistical significance. So far as I am aware, no prior work has identified a statistical test for balance, much less found statistical significance. Further, I identify tests for BR Rep, C-Rep and BR C-Rep, and also for the first time find some evidence of C-Rep and BR C-Rep.

In most existing research, significance is measured in terms of a statistically significant regression coefficient or correlation. Here that is the first stage as reported in Table I.7. Here significance is measured by forecasts. This is a higher hurdle and it is not always cleared in this paper. In Table I.11, Panel A, Column 2, for example, thousands of cases pass the first stage correlation test, but the second stage forecast is insignificant. Forecasting is a step toward decision rules that might beat the averages in competition with human agents.

I.F Conclusion

One who sees glasses as half full will conclude that many of us are good randomizers much of the time. Others of us use representativeness—a non-linear additively non-separable technique that is not half bad. While we can spot their trick using the tools of this paper, their opponents did not consistently find it. One who sees glasses as half empty will see that some of us are exploitable despite an incentive to act otherwise.

I.G Appendix

I.G.1 Kendall's τ

To calculate Kendall's τ (Sprent (1995, p. 169, 176)) in this context: Order path frequencies by the candidate theoretical criterion and assign ranks, call this vector r. Tabulate the empirical path frequencies and assign their ranks, call this vector s, its length n and its i'th element s_i, and order it according to r. Compare each element of s with every subsequent element of s and assign a positive concordance $+1$ if the difference is positive, and a negative discordance -1 if negative. Formally, define:

$$n_c = \sum_{i=1}^{n-1} \sum_{j=i}^{n} I(s_j - s_i > 0)$$

and

$$n_d = \sum_{i=1}^{n-1} \sum_{j=i}^{n} I(s_j - s_i < 0).$$

Then Kendall's $\hat{\tau}$ without tie adjustment is:

$$\hat{\tau} = \frac{2(n_c - n_d)}{n(n-1)}.$$

Assign mid-ranks to ties. Define $D = n(n-1)/2, U = \sum u(u-1)/2$, and $V = \sum v(v-1)/2$ where u and v refer to the number of consecutive ranks in a tie in the r and s vectors respectively. Then Kendall's τ with tie adjustment is:

$$\tau = \frac{2(n_c - n_d)}{\sqrt{(D-U)(D-V)}}.$$

I.G.2 Spearman's ρ

To calculate Spearman's ρ (Sprent (1995, p. 172, 178)) in this context: Compute r and s as for Kendall's τ, then Spearman's $\hat{\rho}$ without tie adjustment is

$$\hat{\rho} = 1 - \frac{\sum_{i=1}^{n}(r_i - s_i)^2}{n(n^2-1)}.$$

Assign mid-ranks to ties and let $C = n(n+1)^2/4$, then Spearman's ρ with tie adjustment is

$$\rho = \frac{\sum_{i=1}^{n} r_i s_i - C}{\sqrt{\sum_{i=1}^{n}(r_i^2 - C)(s_i^2 - C)}}$$

I.G.3 Critical Values

Sprent (1995, Tables 9 and 10) gives some critical values for Kendall's τ and Spearman's ρ conditional on independence. I require critical values for overlapping paths, which are not independent.

I simulated the critical values for the main text using a parametric bootstrap as follows: 800 binary equal-probability pseudo-random outcomes were simulated, and assembled as if they occurred in 20 40-round games. From these paths were extracted and rank correlation statistics calculated. Ten thousand replications of this process were performed. The critical values are order statistics, for example the 80% critical value is the rank correlation statistic that is 2000'th largest.

These critical values are used in the correlations reported in Table I.7. They are also used in the first stage in almost every forecasting model in the paper.

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Chapter II

Investment and Cash Flow in Dynamic Firms Facing Uncertainty and Liquidity Constraints

ABSTRACT

Suppose firms facing conventional production functions, constant returns to scale and symmetric convex adjustment costs take random prices as given and choose capital and labor optimally. In a two-period model, a firm with perfect access to capital markets earns more expected cash flow and expects to invest more after a mean-preserving spread in output prices; a similar liquidity-constrained firm may expect less cash flow and investment after such a change.

II.A Introduction

There is a large literature on the investment decisions of firms that face uncertain prices. In a typical early static model of the firm, profits, cash flow and the marginal value of capital are convex functions of output prices (Oi, (1961)), hence the firm gains from a mean-preserving spread in uncertain prices; uncertainty is "good". The result about uncertainty follows from Jensen's Inequality and is robust. Hartman (1972) (discrete time) and Abel (1983)(continuous time) developed Q Theory dynamic models of investment with convex adjustment costs. In these models too, profits, cash flow and investment rise when uncertainty rises.

By contrast, empirical work (Leahy & Whited (1996)) and intuition suggest that uncertainty hurts firms and cools managements' "animal spirits." Indeed, empiricists tend to ignore the neoclassical prediction that uncertainty is good; see, for example, the literature to which Raddatz (2002) is a recent contribution. Hartman's result, and its endurance, are surprising.

Theorists have proposed several alternatives. The most prominent alternative comes from the real options literature, Dixit & Pindyck (1994). In options, when investment is a sunk cost, uncertainty can be bad, but the result is fragile. The result depends on asymmetric adjustment costs plus declining returns to scale or market power in risk-neutral firms, Caballero (1991). Abel, Dixit, Eberly, and Pindyck (1996) reconcile the real options and Tobin's Q approaches, but their partial equilibrium models respond ambiguously to uncertainty.

Craine (1989) offers a general equilibrium alternative. In Craine's CCAPM model, profits are still a convex function of uncertain prices, but when uncertainty increases, the discount for uncertainty may outweigh the increase in cash flow. Pindyck (1993) has an example in which the firm is indifferent to uncertainty when investment is irreversible, output prices are responsive to industry-wide supply, and shocks are systematic. The implications of these general equilibrium perspectives do not square with the intuition about uncertainty in firms: Uncertainty is perceived to be bad because it hurts profits, cash flow and investment, not because uncertainty is severely discounted by the financial markets.

In short, the literature on investment in firms subject to uncertain output prices has failed to deliver a generally-accepted model consistent with the data or a common sense notion of the effect of uncertainty.

Neoclassical theory also assumes that capital markets are perfect and liquidity constraints are unimportant. But empirical research suggests that liquidity constraints are common (Fazzari, Hubbard & Peterson (1988)) and cash flow is an important factor in how uncertainty affects investment, e.g., Guiso & Parigi (1999). Current investment theory, however, heads in another direction, denying any important independent role for cash flow and liquidity constraints, Abel & Eberly (2001), Gomes (2001). Gomes claims that market value reflects liquidity and so Q Theory captures the empirical salience of cash flow.

In this paper, I relax the assumption that capital markets are perfect and obtain a model consistent with the stylized fact that uncertainty is bad for firms. I solve a discrete time model for $T < \infty$ periods for an unconstrained firm (Proposition 1), and an always-constrained firm (Corollary 2), and then turn to an intermediate case. A mixed firm is constrained when cash is scarce, but unconstrained when cash is plentiful. This is a natural model, but more tedious to solve. Each period doubles the number of possible paths over a binomial tree of constrained and unconstrained stages. Further, if there are more than two periods, precautionary savings may add complications.

Regardless of the assumption about time periods, the liquidity constraint has surprisingly many consequences. The central question is the performance of typical cash flow, investment and capital (collectively "outcomes") in unconstrained, constrained and mixed models, given uncertain prices. One might compare entire distributions of outcomes, but unconstrained firm outcomes come from a different parametric family of distributions than those of the constrained firm. It is unclear how one should compare the distributions directly.

The standard indirect way to compare distributions is through averages, estimated by aggregation over time, perhaps with present value adjustments. Here, however, time averages of outcomes neither stationary nor ergodic for the constrained case because of compounded convex adjustment costs. Time since t_0 matters and the time series averages do not converge. (Hamilton (1994, p. 43-47) reviews stationarity, ergodicity and ensemble averages.)

On the other hand, ensemble averages, averages taken over the distribution at time T, are better-behaved. The ensemble arithmetic average of time T cash flow is a consistent statistic for the central tendency of cash flow in unconstrained firms, being a weighted sum of transformed random variables[1]. But the ensemble arithmetic average is *not* a consistent statistic for the central tendency of outcomes in constrained firms because outcomes are weighted products of transformed random variables. Consequently, the ensemble arithmetic average of constrained firm outcomes do not converge as periods increase without limit; they tend to zero or infinity. Unsurprisingly, there is a convergent measure of ensemble central tendency of outcomes in constrained firms: it is the ensemble geometric average or $e^{E[\log \tilde{X}_T]}$ where $\tilde{X}_T$ is a period

[1]The other outcomes, investment and capital, are deterministic

T random outcome, cumulative cash flow, investment or capital. (Hereafter "ensemble" is assumed.)

The main result of the paper is that arithmetic average outcomes rise with mean-preserving spreads in uncertain output prices in unconstrained firms but geometric average outcomes may fall with mean-preserving spreads in uncertain output prices in mixed and constrained firms.

The result depends on the different measures of central tendency. For the maintained assumptions, outcomes in any one period are positive convex functions of output prices. An arithmetic average is linear, so the convexity carries over to an unconstrained dynamic model and uncertainty is good. The geometric average is a concave exponential function. The exponential concavity reduces or reverses convexity, so outcomes may be concave functions of output prices, making uncertainty bad for the constrained firm. Propositions 3 and 4 formalize these claims and their limitations.

Firms that are constrained at some times and unconstrained at others present a mixed case; their central tendency is a probability weighted arithmetic average of arithmetic (for unconstrained states) and geometric (for constrained states) averages.

In section 2, I propose and solve a model that nests three cases: unconstrained firms, constrained firms, and mixed firms. Although the argument depends heavily on functional form assumptions, I show that under certain conditions, uncertainty is good for outcomes in unconstrained firms, bad for constrained ones, and the character of mixed firms depends on how closely they resemble constrained or unconstrained firms. In section 3 the result is illustrated with simulations. In the discussion section 4, I review the static part of the model, showing three ways that mean-preserving spreads in output prices affect results. Then I consider whether liquidity constraints are reflected in institutional facts and whether they are endogenous or exogenous. Conclusions follow. Appendix A has proofs.

II.B The Model

We start with a firm that is unconstrained and entirely neoclassical; it can borrow and lend in perfect capital markets. A second firm is identical, except for the liquidity constraint. The constraint could take many forms. For simplicity I assume it takes the strongest possible form: The constrained firm cannot borrow at all.

II.B.1 Setup

The firm's optimization problem can be described in terms of its production function, equations of motion for its state variables, capital K_t and cash balances M_t, and their initial values K_0 and M_0. The production function is Cobb-Douglas with constant returns. The Cobb-Douglas functional form assumption is strong, important for tractability, and examined below when more of the pieces are in place. The production function is

$$Q_t = L_t^{\beta} K_t^{1-\beta}, \beta \in (0,1);$$

where $Q_t \geq 0$ is output in period t; $K_t \geq 0$ is capital and $L_t \geq 0$ is labor.

The equation of motion for capital is

$$K_{t+1} = (1-\delta)K_t + I_t,$$

where δ is the depreciation rate and I_t is period t capital investment in physical units. I_t is available for production in period $t+1$.

The equation of motion for cash balances is built up from input and output costs. The firm pays wages w for labor. The capital adjustment cost function is convex, symmetric and differentiable, specifically:

$$C(I_t, q) = (I_t q)^\gamma, \quad q > 0, \gamma > 1.$$

The firm can save funds it does not use in operations at gross interest rate R. The unconstrained firm can borrow at rate R without limit and M_t can be negative. All parameters are deterministic except the output price $\tilde{p}$, which is i.i.d. with probability measure $f(p)$ having support in R_+, cumulative distribution function $F(p)$ and finite variance. The firm knows p_t in time to choose L_t and I_t[2]. Sometimes hereafter the tilde ˜ is suppressed. Of course, taking price as the only random variable is a simplification. Firms experience many sources of risk, including uncertain technological progress and uncertain factor prices. Here $\tilde{p}$ is a proxy for all these risks. It may be a poor proxy for some of them. The equation of motion for money is

$$M_{t+1} = p_t Q_t + RM_t - C(I_t, q) - wL_t.$$

The firm is risk-neutral. Its objective is to maximize expected cash balances at period T.

The firm's problem is, therefore,

$$\begin{array}{ll} \max_{L_t, I_t} & p_T L_T^\beta K_T^{1-\beta} + RM_T - wL_T \\ \text{subject to} & K_{t+1} = (1-\delta)K_t + I_t \\ & M_{t+1} = p_t L_t^\beta K_t^{1-\beta} + RM_t - (I_t q)^\gamma - wL_t \\ & L_t, K_t \geq 0 \\ & K_0 \text{ and } M_0 \text{ given, } t = 0, 1; T = 2. \end{array}$$

The constrained firm is identical, except the constrained firm cannot borrow, $M_t \geq 0, \forall t$. In particular, $M_0 \geq 0$, so the constrained firm does not start out in debt.

[2]More formally, suppose $\{\mathcal{P}_t\}$ is a filtration, a sequence of σ-fields $\mathcal{P}_t \subset \mathcal{P}$ such that $\mathcal{P}_{t-1} \subset \mathcal{P}_t, \forall t$, and that $\{\mathcal{P}_t\}$ is the σ-field generated by $\{\tilde{p}_i\}_{i=0}^{t}, p_t > 0$. Suppose also the filtration $\mathcal{P}$ is adapted to $\{p_t\}$, *i.e.*, $\{p_t\}$ is a sequence of random scalars such that p_t is measurable with respect to $\mathcal{P}_t$.

The Cobb-Douglas production function favors tractability and intuition over verisimilitude. Because the Cobb-Douglas production function incorporates Inada conditions, cash flow from operations is always positive. This matters to the constrained firm. Since cash flow from operations is positive and there is no debt, the constrained firm does not risk bankruptcy in the equity sense, inability to pay its obligations as they become due. On the other hand, the constrained firm may be unable to replace depreciating capital out of cash flow, and so it can shrivel to trivial size, even when its unconstrained twin finds it optimal to expand. This set of assumptions avoids confusing issues. The constraint on borrowing, not fixed costs or risks of bankruptcy, drives the results.

The solution technique applies Bellman's principal of optimality: The optimal policy must be optimal from any arbitrary point forward. Accordingly, the solution strategy is to start at the final period and work backward in time. I start with a one-period firm and elaborate. Proofs are in Appendix A.

II.B.2 Two Periods

For the two-period unconstrained model, start with the one-period result (Lemma 1) and consider optimization in the previous period. $E_0[M_2|U_0]$ is the expectation taken at $t = 0$ of money at $t = 2$ given no constraint at $t = 0$. $E_0[M_2|C_0]$ is the same expectation conditional on a binding constraint at $t = 0$.

Lemma 2 (Two-period Unconstrained Firm). *Expected terminal cash balances for a 2-period unconstrained firm are:*

$$\begin{aligned} E_0[M_2|U_0] &= R(\gamma-1)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\ &+ \left((R+1-\delta)AE[p^{\frac{1}{1-\beta}}]\right)K_0 + R^2 M_0. \end{aligned}$$

Terminal capital is:

$$K_1 = (1-\delta)K_0 + \frac{1}{q}\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{1}{\gamma-1}}.$$

Cash investment is:

$$I_0 = \left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}}.$$

The result can be interpreted as follows: cash flow is a sum of four elements: (1) cash flow in period zero on initial capital with interest, $RAE[p^{\frac{1}{1-\beta}}]K_0$, plus (2) cash flow in period one on depreciated initial capital, $(1-\delta)AE[p^{\frac{1}{1-\beta}}]K_0$, plus (3) cash flow on investment made in period zero, available in period one, $R(\gamma-1)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}}$, plus (4) a return on initial cash balances R^2M_0. Investment and terminal period capital are deterministic.

The T-period case is a straightforward generalization, see Appendix A Proposition 1.

Turn now to the mixed case. The firm has two regimes. If cash is low, the firm is constrained; if cash is high it is unconstrained. Find p_0^*, the price which separates these regimes, by setting realized cash balance equal to target investment and solving for the output price.

Lemma 3 (Two period p_0^*).

$$p_0^* = \left(\frac{\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} - RM_0}{AK_0}\right)^{1-\beta}.$$

Note that p_0^* depends upon the number of periods in the model. The p_0^* in Lemma 3 is for the first of two periods.

If $p_0 < p_0^*$ then the firm is in the constrained regime. Since L_t^* maximizes cash flow and is always feasible (since cash flow is strictly positive due to Inada conditions), the constrained firm can choose L_t just as the unconstrained firm does. The marginal return from investment exceeds the return from saving at all feasible levels of investment so the firm invests all available cash.

Proposition 2 (Two-period Firm). *Expected terminal cash balances for a 2-period firm are:*

$$E_0[M_2] = E_0[M_2|U_0]Pr[U_0] + E_0[M_2|C_0]Pr[C_0]$$

where $E_0[M_2|U_0]Pr[U_0] =$

$$(1 - F[p_0^*]) \times \left(R(\gamma - 1) \left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q} \right)^{\frac{\gamma}{\gamma-1}} + (1 - \delta + R)AE[p^{\frac{1}{1-\beta}}]K_0 + R^2 M_0 \right)$$

and $E_0[M_2|C_0]Pr[C_0] =$

$$F[p_0^*] \exp \left[\int_0^{\infty} \log \left(Ap_T^{\frac{1}{1-\beta}} \right) dp_1 + \int_0^{p_0^*} \log \left(\frac{1}{q} \left(AK_0 p_0^{\frac{1}{1-\beta}} + RM_0 \right)^{\frac{1}{\gamma}} + (1 - \delta)K_0 \right) dp_0 \right].$$

Cash investment is:

$$\begin{aligned} E_0[I_0] &= (1 - F[p_0^*]) \left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q} \right)^{\frac{\gamma}{\gamma-1}} \\ &+ \exp \int_0^{p_0^*} \log \left(AK_0 p_0^{\frac{1}{1-\beta}} + RM_0 \right) dp_0 \\ E_0[I_1] &= 0. \end{aligned}$$

If $Pr[p_0 > p_0^*] = 1$ the firm is unconstrained and Lemma 2 is recovered. If $Pr[p_0 > p_0^*] = 0$, $M_0 = 0$ and $\delta = 1$ then the firm is always constrained and capital short-lived. Think of always constrained firms as small firms whose target investment is always large relative to cash balances and cash flow. Hence:

Corollary 1. *Realized money for a 2-period always-constrained firm is:*

$$M_2 = \frac{A^{\frac{\gamma+1}{\gamma}} K_0^{\frac{1}{\gamma}}}{q} p_1^{\frac{1}{1-\beta}} p_0^{\frac{1}{\gamma(1-\beta)}},$$

while its expected terminal cash balance is:

$$E_0[M_2] = \frac{A^{\frac{\gamma+1}{\gamma}} K_0^{\frac{1}{\gamma}}}{q} \exp\left(\frac{1+\gamma}{\gamma(1-\beta)} E[\ln(p)]\right).$$

Investment is zero in period 1 and in period zero

$$I_0 = A K_0 p_0^{\frac{1}{1-\beta}},$$

while expected investment is:

$$E_0[I_0] = A K_0 e^{\frac{1}{1-\beta} E[\log p]}$$

$$E_0[I_1] = 0$$

A T-period version of this model can be obtained by induction:

Corollary 2. *Terminal cash balances M_{T+2} for a T-period ($T > 1$) always-constrained firm are:*

$$M_{T+1} = \frac{A^{\sum_{i=1}^{T} \frac{\gamma^{i-1}}{\gamma^{T-1}}} K_0^{\frac{1}{\gamma^{T-1}}}}{q^{\sum_{i=1}^{T-1} \frac{\gamma^{i-1}}{\gamma^{T-2}}}} \prod_{i=1}^{T} (p_{T-i})^{\frac{1}{\gamma^{i-1}(1-\beta)}},$$

while expected terminal cash balances are:

$$E_0[M_{T+1}] = \frac{A^{\sum_{i=1}^{T} \frac{\gamma^{i-1}}{\gamma^{T-1}}} K_0^{\frac{1}{\gamma^{T-1}}}}{q^{\sum_{i=1}^{T-1} \frac{\gamma^{i-1}}{\gamma^{T-2}}}} \exp\left(\frac{E[\log(p)]}{(1-\beta)\sum_{i=1}^{T} \gamma^{i-1}}\right).$$

Investment just the previous period's cash flow except in the final period, when it is zero.

In the always-constrained firm, there is only one source of capital. That source is capital from cash flow invested in the previous period. Any older capital has depreciated, and there is no accumulated cash balance. Note that the difference between models of differing length of time T is confined to functions of γ. This makes sense: Adjustment costs are incurred over and over; the longer the time period the more their compounded impact.

II.B.3 Mean-preserving Spreads

It is possible to analytically examine the effect of mean-preserving spreads in the unconstrained and constrained models. Given Cobb-Douglas assumptions, in the unconstrained model investment and cash flow are a weighted sum of increasing functions of random variables raised to a power greater than one, so mean-preserving spreads in prices increase the expectation. In the constrained firm investment and cash flow are a weighted product of increasing functions of random variables transformed by a function. The convergent expectation is a geometric mean. Since geometric means are concave they partially or completely offset the convexity arising from the static part of the model. The consequences are spelled-out in two easy propositions:

Proposition 3 (Additive Mean-preserving Spreads). *If an expected value is a linear combination of (1) positive random variables raised to a power greater than one, (2) positive increasing functions of expectations of positive random variables raised to a power greater than one, and (3) constants, then the expected value is increasing in a mean-preserving spread of its random variables.*

Proposition 4 (Multiplicative Mean-preserving Spread). *If an expected value is a positive increasing product of random variables transformed by twice differentiable functions* $g_i(\cdot)$ *such that* $g_i' g_i' > g_i'' g_i, \forall g_i$, *then the expected value is decreasing in a mean-preserving spread of its random variables.*

Proposition 3 applies to $E_0[M_2|U_0]$, and it can be easily seen if one simplifies the deterministic parts, replacing them with constants c_i.

$$E[M_2|U_0] = c_1(E[p^{\frac{1}{1-\beta}}])^{\frac{\gamma}{\gamma-1}} + c_2 E[p^{\frac{1}{1-\beta}}] + c_3.$$

The expectation is a linear combination of three terms. The first term is a increasing function of an expectation of the random variable price, the price being transformed by positive power $\frac{1}{1-\beta}$. The second term is an linear function of the random variable price transformed by raising it to the positive power $\frac{1}{1-\beta}$. The third term is a constant. Accordingly, the conditions of the proposition are met for $\beta > 0$ and $E[M_2|U_0]$ is increasing in mean-preserving spreads in random prices.

The Proposition 4 applies ambiguously to $E_0[M_2|C_0]$. Simplifying as before:

$$E[M_2|C_0] = E[c_1 p_1^{\frac{1}{1-\beta}}(c_2(c_3 p_0^{\frac{1}{1-\beta}} + c_4)^{\frac{1}{\gamma}} + c_5)].$$

$$E[M_2|C_0] = \exp\left(c_1 E[\log(p_1^{\frac{1}{1-\beta}})] + E[\log(c_2(c_3 p_0^{\frac{1}{1-\beta}} + c_4)^{\frac{1}{\gamma}} + c_5)]\right).$$

There are two expectations on the right-hand side. The first, $\log(p_1^{\frac{1}{1-\beta}})$ is concave, but the second, $\log(c_2(c_3 p_0^{\frac{1}{1-\beta}} + c_4)^{\frac{1}{\gamma}} + c_5)$, is ambiguous; it may be convex for low values of p and concave for high values of p. To determine analytically how a constrained firm responds to uncertainty an assumption about the price distribution may be necessary.

A mean-preserving spread increases p_0^* as well. If the firm is mixed, the effect of uncertainty depends on whether the concavity dominates convexity in the constrained regime, how heavily the two regimes are weighted and how much p_0^* moves. In the next section, simulations illuminate these points.

II.C Simulations on Constraints and Mean-Preserving Spreads

In this section simulations illustrate how mean-preserving spreads affect expected outcomes (cash balances, capital and investment) in unconstrained, constrained and mixed models.

Parameters defined in Section 2.1 have the values, suggested as stylized facts, given in Table II.1. Prices have either a Bernoulli or a lognormal

Table II.1: Benchmark Parameter Values

Description	Parameter	Value
Labor exponent	β	2/3
Wage rate	w	1
Gross interest rate	R	1.05
Depreciation rate	δ	0.1
Adjustment cost exponent	γ	2
Adjustment cost shifter	q	1
Initial capital	K_0	1
Initial cash balance	M_0	0

distribution, detailed later. Being known to the firm, target investment was calculated from population parameters. One hundred thousand pairs of prices were generated for each experiment. The same seed was used for each experiment. All the simulations were run in Mathematica 4.1.

Expected values were obtained. Specifically, for the unconstrained regime, investment is a constant, equation (II.5); the algorithm calculated terminal cash balances as an arithmetic average of simulants using equation (II.7). For the constrained case, the algorithm calculated investment and cash balances as geometric averages using $AK_0p_0^{\frac{1}{1-\beta}} + RM_0$ (but recall that $M_0 = 0$) and equation (II.10). For the mixed case, the algorithm used the unconstrained procedure for all simulants experiencing $p_0 > p_0^*$ and otherwise the constrained procedure. These averages were themselves arithmetically averaged according to their sample frequencies, to find $E_0[M_2]$ from Proposition 2.

I considered two distributional assumptions, Bernoulli prices and lognormal prices. Suppose prices have a Bernoulli distribution; prices are either low 3.9 or high 4.1 with equal probability. Formally,

$$p_{base} = \{3.9, 1/2; 4.1, 1/2\}.$$

Over two periods there are four equally likely realizations: {(3.9, 3.9), (3.9,

Table II.2: Bernoulli Prices Experiment

Distribution [mean, s.d.]	Unconstrained	Constrained	Mixed
Base $[4.0002, 0.0999998]$			
Population p_0^*			5.17
$\Pr[p < p^*]$	0	1	1
Investment	20.46	9.473	9.473
Cash Balance	40.02	37.69	37.69
Spread $[4.0018, 0.899998]$			
Population p_0^*			5.67
$\Pr[p < p^*]$	0	1	1
Investment	27.05	8.775	8.775
Cash Balance	49.83	34.31	34.31

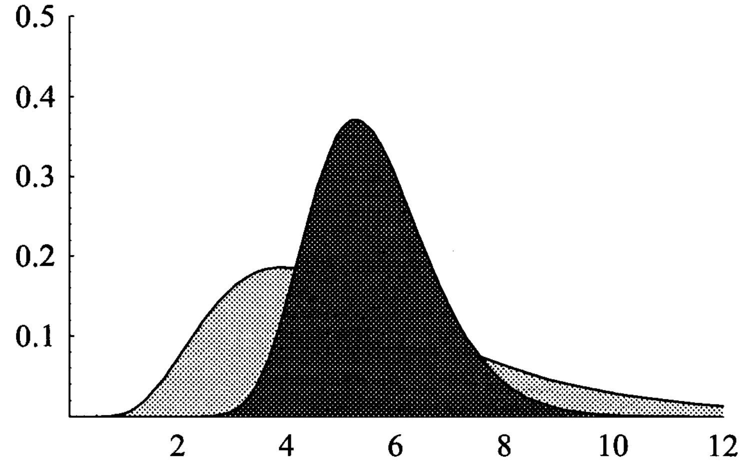
4.1), (4.1,3.9),(4.1,4.1)}. For a mean-preserving spread consider

$$p_{spread} = \{3.1, 1/2; 4.9, 1/2\}.$$

Table II.2 reports results. For the unconstrained model, investment rose after the mean-preserving spread in prices from 20.46 to 27.04 and cash balances rose from 40.02 to 49.83, while for the constrained case, investment fell after a mean-preserving spread in prices from 9.473 to 8.775 and cash balances fell from 37.79 to 34.31. For these parametric assumptions, the mixed case was the same as the constrained case because the constraint was binding after both high and low initial period prices; p_0^* was 5.15 for the base case and 5.67 for the mean-preserving spread.

A continuous distribution of output prices allows for a more interesting mixed case, and financial prices are thought to have approximately a log-normal distribution. Consider $p_1 \sim LN[1.7, 0.2]$ and $p_2 \sim LN[1.6, 0.489898]$. Both have population expected value 5.5843, the second is a mean-preserving spread from the first. Figure II.1 illustrates these distributions. Table II.3 reports results. For the unconstrained model, investment rose after the mean-

Figure II.1: LN[1.7, 0.2] and its Mean-Preserving Spread LN[1.6, 0.49]



preserving spread in prices from 257.297 to 637.172 and cash flow rose from 257.297 to 756.282, while for the unconstrained case, investment fell after a mean-preserving spread in prices from 24.3451 to 18.0842 and cash flow fell from 142.432 to 96.1857. For the mixed firm the constraint was binding for both the base case and the mean-preserving spread more than 99% of the time, but results were nevertheless distinct from the constrained model. For the mixed model, investment fell after the mean-preserving spread from 24.3768 to 22.3395 and cash flow fell from 142.478 to 109.237.

The unconstrained results are comparable to those of Lee & Shin (2000), who vary β in an unconstrained model.

II.D Discussion

Section 2 was an analysis of how unconstrained and unconstrained firms respond differently to mean-preserving spreads in prices. In section 3 these results were supported by simulations in which unconstrained firms gained from uncertainty and constrained firms suffered from uncertainty. Here

Table II.3: Lognormal Price Experiment

Model [sample mean, sample s.d.]	Unconstrained	Constrained	Mixed
Base [5.58473, 1.12569]			
Population p_0^*			10.9011
$\Pr[p < p^*]$	0	1	0.99971
Investment	191.913	24.3451	24.3768
Cash Balance	257.297	142.432	142.478
Spread [5.58324, 2.89903]			
Population p_0^*			16.2625
$\Pr[p < p^*]$	0	1	0.99238
Investment	637.172	18.0842	22.3395
Cash Balance	756.282	96.1857	109.237

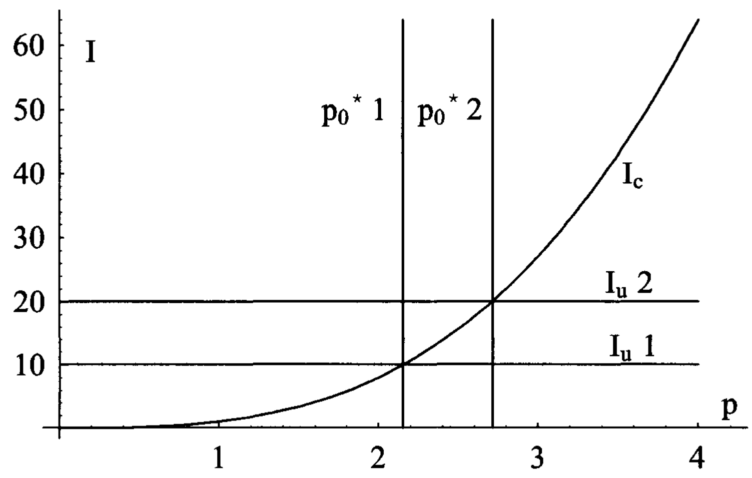
I consider some related issues: the effect of uncertainty from a static perspective, and connections between the model and institutional facts about firms and macroeconomics.

II.D.1 Effects of Uncertainty from a Static Perspective

While investment is a flow and so inherently dynamic, our neoclassical intuitions about firm behavior are grounded in a static perspective. Consider Figure II.2, where realized price is on the horizontal axis and investment on the vertical. Investment for unconstrained firms I_u depends on the price distribution, but does not depend on price realizations. A mean-preserving spread in prices increases target investment from, say, $I_u(1)$ to $I_u(2)$. Investment for constrained firms I_c depends on realized prices, but not on the price distribution. For a mixed firm, investment is the lesser of I_u and I_c, and p_0^*, the price for which $I_u = I_c$, is important.

In comparative statics, there are two ways mean-preserving spreads can increase investment, by increasing I_u directly and because of the convexity of I_c with respect to price. There is one way mean-preserving spreads can

Figure II.2: Mean-Preserving Spreads and Investment in Cross-Sections



decrease investment, due to the concave kink at $p = p_0^*$.

All these comparative static effects may be dominated by dynamics, the way that outcomes in any given period combine with outcomes in other periods. Liquidity constraints change the dynamics from an additive process to a multiplicative one.

II.D.2 Liquidity Constraints Are Realistic Assumptions

Liquidity constraints offer a solution to Hartman's puzzle. That is interesting, but liquidity constraints are more than a technical band-aid for an ailing theory. Empirical evidence and theory abound on liquidity constraints. It is no surprise that finance is constrained in the developing world, (Rajan & Zingales (1998)), but liquidity constraints bind U.S. consumers, (Gross & Souleles (2002)), cause under-capitalization of U.S. startups, (Evans & Jovanovic (1989)) and increase the likelihood of business failure, (Holtz-Eakin, Joulfaian & Rosen (1994)). A lemons problem creates an exogenous constraint on firm finance. The incentive to send a signal may generate endogenous constraints, *e.g.*, Greenwald & Stiglitz (1990).

A good example of an unconstrained firm is a commercial bank. A bank makes a market in liquidity, borrows daily from and lends daily to its customers. It actively manages balance sheet leverage; however, even banks show the limits of the unconstrained model. While banks manage their debt on a daily, or even hourly basis, they are bound by reserve requirements and do not constantly adjust their equity.

The canonical constrained firm may be Messrs. Hewlett and Packard working nights in a now-famous garage. They started Hewlett Packard Company with unmeasurable opportunity, and financing tied to that which can be measured, which is to say no financing. Or consider Michael Dell paying his college tuition by manufacturing computers in his dorm, between classes. Their asset was their talent, their human capital, which makes poor collateral both because it is difficult to assess and because it is difficult to repossess.

These are illustrations of firms that anticipate high expected gains coupled with great uncertainty. They may yield a low expected growth rate. The sorts of firms likely to be exogenously constrained are precisely those firms most likely to have high expected future profits subject to great uncertainty. Indeed, it is the blend of large and uncertain prospects that makes them appealing, but not bankable, opportunities.

Constraints can be endogenous too. Perhaps an investor in an unconstrained firm could replicate a constrained firm's return by reinvesting unconstrained firm dividends as they are paid out. If the constraint distorts the decisions of the constrained firm, all other things equal, the investor can do better than replicate the constrained firm's returns. But the investor may not be able to replicate the constrained firm if there is a boundary between the investor and the firm, say a tax wedge imposed on distributions or information asymmetries. Because of these frictions it may be optimal to have the compounding occur within the firm instead of with reinvestment by the investor. Frictions can make a firm act as if it were constrained.

At some point, the prospects of H-P and Dell Computer became tangible enough to find favor in the financial markets. They were able to borrow working capital and issue equity. Yet the modern H-P and Dell, along with thousands of high-technology peers, still finance sparingly. Some such companies, Intel and Cisco come to mind, have tens of billions of dollars in cash, but negligible dividends. They do not need these funds as precautionary savings since they exceed any plausible liquidity requirement. Their war chests also exceed any plausible needs to fund acquisitions. Surely they could make distributions, and they could raise additional debt or equity with relative ease. They could manage their balance sheets, but instead have cash and equivalents commensurate with what their business yields them. Their investors are paid through capital gains in the value of equity. For these firms, constraints may be the endogenous outcome of an optimization decision.

Shakespeare's character Polonius, in saying "neither a borrower nor a lender be", makes endogenous constraints a moral choice. If we set him aside and limit ourselves to conventional economists' justifications for endogenous constraints, many remain beyond my stylized examples. Because it may be difficult to get liquidity into companies, they tend to hoard cash, favor internal finance, and avoid dividends, except where dividends serve as signals. For the present purpose, the enormous and subtle literature on these issues is background. What matters is that the frictions are so important that liquidity constraints are a valid assumption for some firms.

II.D.3 History Dependence

Constrained firms are profoundly history dependent. A poor price may forever hobble a constrained firm. Though the firms modelled here never go bankrupt, constrained firms can still suffer extreme adversity while unconstrained firms can base their plans on averages, and may borrow their way around bad times.

Price variations push the firm from one regime to the other with a probability that changes over time to favor the unconstrained regime if early realizations were favorable. While the future of the firm is captured by its state variables, cash and capital depend on the path taken. In Corollary 2, prices are raised to a power that falls as t increases. Early price realizations carry more weight in terminal value than later ones. A firm with the good fortune to get good prices early will do better than one with the same price distribution, but bad luck in *when* the good times roll.

Liquidity constraints are important to the credit view of monetary policy. This paper may be viewed as work on the micro foundations of Gertler & Gilchrist (1994). Gertler and Gilchrist argue that small firms have fewer and more expensive financing alternatives. Information asymmetries render small firms captives of small banks. In the credit view, monetary policy works because a tightening of policy squeezes small banks, who in turn squeeze their customers. Small firms cannot be totally constrained, after all they are small bank customers. but constraints may describe small firms better than perfect financial markets, and constraints may describe small firms better than it describes large firms.

II.E Conclusion

A business person might think that one of the easier, well-defined questions she might ask an economist is "How do I divide my fixed resources between a wage bill for current production and a capital budget, subject to uncertain future prices?"

An open-ended approach to this open-ended question depends upon the character of the firm. It depends upon the length of time between optimization decisions, how many time periods occur, how gains aggregate within time periods, how gains aggregate across time periods and whether re-optimization is financially constrained. If the time period may vary, the extreme case of an unconstrained firm is one that re-optimizes its financing continuously, the implicit assumption of Abel (1983). The extreme constrained firm is one whose period between financing re-optimizations is infinite. Here, however, these modelling choices are taken to be beyond the scope of the analysis, and all save one are fixed. The variable under study is liquidity constraints.

Subject to the maintained assumptions, especially the short two-period horizon and the 'no lose' feature of the Cobb-Douglas production function, the answer is that the firm sets labor to its unconstrained optimum and invests all remaining cash so long as an unconstrained firm would.

The assumption that constrained firms have no access to financing is strong. Perhaps most have some form of imperfect access to finance. Nevertheless strong liquidity constraints yield a relatively simple model in which uncertainty may be bad for firms in that uncertainty may reduce investment and cash flow. Constraints also capture important institutional facts. Liquidity constraints belong in the mix of modelling elements used to analyze firms, along with market power, returns to scale, indivisibility, irreversibility and asymmetries in adjustment costs.

There is a lot of work to be done to show that important variations of the model are robust: (1) Here unconstrained firm capital is chosen to optimize returns when there is only one period left. Moving to many periods or continuous time in an unconstrained firm is an minor extension of the existing work of others, but the constrained and mixed models are more challenging extensions. (2) Allow negative cash flow, where, for the constrained firm, there is a risk of bankruptcy. (3) To make optimal scale well-defined, some sort of limitation on expansion is needed. Here is it convex adjustment costs. It may be useful to move this source of determinacy to returns to scale, market power or risk aversion. (4) Here output prices are the sole random variable, but other prices vary and technological progress is uncertain. (5) Here access to finance is perfect, or absent. The model could be extended to allow for intermediate cases of various kinds. (6) This paper is a theory exercise. Testable propositions beyond the stylized fact that uncertainty is bad need to be derived and tested.

If this model is accepted as a plausible approach to investment and the firm, it may find application in macroeconomics and industrial organization. It also has a potential parallel in consumption theory.

II.F Appendix

Lemma 1 (One-period Firm). *Expected cash flow for a 1-period firm is:*

$$E_0[M_1] = AK_0 E[p^{\frac{1}{1-\beta}}] + RM_0 \qquad \text{(II.1)}$$

and investment is zero; A is a function of parameters.

Proof. In any time period t, capital is fixed at K_t so the firm's cash at the beginning of period $t+1$ is just its cash equation,

$$M_{t+1} = p_t L_t^{\beta} K_t^{1-\beta} + RM_t - C(I_t, q) - wL_t.$$

To maximize cash, set the derivative of the choice variable L_t equal to zero and rearrange,

$$L_t^* = K_t \left(\frac{w}{\beta p_t}\right)^{\frac{1}{\beta-1}}.$$

Substitute L_t^* back into M_{t+1} and simplify,

$$\begin{aligned} M_{t+1} &= (1-\beta)(\frac{w}{\beta})^{\frac{\beta}{\beta-1}} K_t p_t^{\frac{1}{1-\beta}} + RM_t - C(I_t, q) \\ &= AK_t p_t^{\frac{1}{1-\beta}} + RM_t - C(I_t, q), \qquad \text{(II.2)} \end{aligned}$$

where $A = (1-\beta)(\frac{w}{\beta})^{\frac{\beta}{\beta-1}}$. Equation (II.2) is the labor-optimized cash equation of the firm.

Optimal investment is zero in the last period and here the first period is the last period, so adjustment costs are zero.

$$M_{T+1} = AK_T p_T^{\frac{1}{1-\beta}} + RM_T. \qquad \text{(II.3)}$$

Let $E_t[\cdot]$ be an expectation taken with information known at time t. Set $T = 0$ and take an expectation. The result follows if $E[p^{\frac{1}{1-\beta}}] < \infty$, *see*, White (2001, p. 32). □

Lemma 2 (Two-period Unconstrained Firm). *Expected cash flow for a 2-period unconstrained firm is:*

$$\begin{aligned} E_0[M_2|U_0] &= R(\gamma-1)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\ &+ (1-\delta+R)E[p^{\frac{1}{1-\beta}}]AK_0 + R^2M_0, \end{aligned} \tag{II.4}$$

Cash investment is:

$$I_0 = \left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}}. \tag{II.5}$$

Proof. Equation (II.3) uses optimal behavior in the last period T to determine expected cash after that period ends. Working backward, treat M_T as endogenous, determined in $T-1$ and K_T as a control. Begin by eliminating M_T using (II.2) (set $t+1$ equal to T) and then apply the definition of investment.

$$\begin{aligned} M_{T+1} &= AK_Tp_T^{\frac{1}{1-\beta}} + R\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}} + RM_{T-1} - (I_{T-1}q)^{\gamma}\right) \\ &= AK_Tp_T^{\frac{1}{1-\beta}} \\ &+ R\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}} + RM_{T-1} - ((K_T-(1-\delta)K_{T-1})q)^{\gamma}\right) \end{aligned} \tag{II.6}$$

At $T-1$, p_T is unknown. However, its distribution is known, and the firm is risk neutral, so an expectation taken at $T-1$ converges if $E[p^{\frac{1}{1-\beta}}] < \infty$,:

$$\begin{aligned} E_{T-1}[M_{T+1}] &= AK_TE[p_T^{\frac{1}{1-\beta}}] \\ &+ R\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}} + RM_{T-1} - ((K_T-(1-\delta)K_{T-1})q)^{\gamma}\right) \end{aligned}$$

Take a derivative with respect to the control K_T and solve for K_T^*,

$$K_T^* = (1-\delta)K_{T-1} + \frac{1}{q}\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{1}{\gamma-1}},$$

and substitute it back into (II.6) and simplify,

$$
\begin{aligned}
M_{T+1} &= R\left(\gamma\frac{p_T^{\frac{1}{1-\beta}}}{E[p^{\frac{1}{1-\beta}}]} - 1\right)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\
&+ \left(p_T^{\frac{1}{1-\beta}}(1-\delta) + p_{T-1}^{\frac{1}{1-\beta}}R\right)AK_{T-1} + R^2 M_{T-1} \qquad \text{(II.7)}
\end{aligned}
$$

The random variables already reduced to expected values in (II.7) are used to calculate target investment; they are deterministic. At $T-1$, p_T is unknown, but we can take an expectation.

$$
\begin{aligned}
E_{T-1}[M_{T+1}] &= R(\gamma-1)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\
&+ \left((1-\delta)E[p^{\frac{1}{1-\beta}}] + Rp_{T-1}^{\frac{1}{1-\beta}}\right)AK_{T-1} + R^2 M_{T-1}
\end{aligned}
$$

Average over p_{T-1}, set $T = 1$ and the result follows where U_0 recalls the assumption that the firm is unconstrained in period zero. □

Proposition 1 (Unconstrained Firm). *Expected value for a T-period unconstrained firm is:*

$$
\begin{aligned}
E[M_{T+1}|U_t \forall t] &= \sum_{i=1}^{T} R^{T-i+1}(\gamma-1)\left(\frac{X[T-i]AE[p^{\frac{1}{1-\beta}}]}{R^{T-i+1}\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\
&+ X[T]AE[p^{\frac{1}{1-\beta}}]K_0 + R^{T+1}M_0,
\end{aligned}
$$

where

$$
X[j] = \sum_{k=0}^{j} R^k (1-\delta)^{j-k}.
$$

Further, cash investment in period t is

$$
I_t = \left(\frac{X[T-t]AE[p^{\frac{1}{1-\beta}}]}{R^{T-t+1}\gamma q}\right)^{\frac{\gamma}{\gamma-1}}.
$$

Proof. Substitute for M_{T-1} in (II.7) using (II.2) and the definition of investment:

$$\begin{aligned} M_{T+1} &= R\left(\gamma\frac{p_T^{\frac{1}{1-\beta}}}{E[p^{\frac{1}{1-\beta}}]}-1\right)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\ &+ \left(p_T^{\frac{1}{1-\beta}}(1-\delta)+p_{T-1}^{\frac{1}{1-\beta}}R\right)AK_{T-1} \\ &+ R^2\left(AK_{T-2}p_{T-2}^{\frac{1}{1-\beta}}+RM_{T-2}-((K_{T-1}-(1-\delta)K_{T-2})q)^{\gamma}\right) \end{aligned} \quad \text{(II.8)}$$

K_{T-1} is determined at $T-2$, when p_T and p_{T-1} are unknown, but we can take expectations:

$$\begin{aligned} E_{T-2}[M_{T+1}] &= R(\gamma-1)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\ &+ (1-\delta+R)\,AE[p^{\frac{1}{1-\beta}}]K_{T-1} \\ &+ R^2\left(AK_{T-2}p_{T-2}^{\frac{1}{1-\beta}}+RM_{T-2}-((K_{T-1}-(1-\delta)K_{T-2})q)^{\gamma}\right). \end{aligned}$$

Take the derivative with respect to K_{T-1} and solve for K^*_{T-1},

$$K^*_{T-1} = (1-\delta)K_{T-2}+\frac{1}{q}\left(\frac{(1-\delta+R)AE[p^{\frac{1}{1-\beta}}]}{R^2\gamma q}\right)^{\frac{1}{\gamma-1}}$$

and substitute into $(II.8)$:

$$\begin{aligned} M_{T+1} &= R\left(\gamma\frac{p_T^{\frac{1}{1-\beta}}}{E[p^{\frac{1}{1-\beta}}]}-1\right)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\ &+ R^2\left(\frac{(p_T^{\frac{1}{1-\beta}}(1-\delta)+p_{T-1}^{\frac{1}{1-\beta}}R)\gamma}{(1-\delta+R)E[p^{\frac{1}{1-\beta}}]}-1\right)\left(\frac{(1-\delta+R)AE[p^{\frac{1}{1-\beta}}]}{R^2\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\ &+ \left((p_T^{\frac{1}{1-\beta}}(1-\delta)+p_{T-1}^{\frac{1}{1-\beta}}R)(1-\delta)+R^2Ap_{T-2}^{\frac{1}{1-\beta}}\right)AK_{T-2} \\ &+ R^3M_{T-2}, \end{aligned} \quad \text{(II.9)}$$

where K_{T-2} and M_{T-2} are state variables. At time $t = T-3$, later prices are unknown; take an expectation and with i.i.d prices the expression simplifies,

$$E_{T-3}[M_{T+1}] = R(\gamma-1)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}}$$

$$
\begin{aligned}
&+ \quad R^2(\gamma-1)\left(\frac{(1-\delta+R)AE[p^{\frac{1}{1-\beta}}]}{R^2\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\
&+ \quad \left((1-\delta+R)(1-\delta)+R^2\right)AE[p^{\frac{1}{1-\beta}}]K_{T-2}+R^3M_{T-2},
\end{aligned}
$$

By induction on T, for any T-period model with $0 < T < \infty$ (for $T = 0$ the summation term vanishes),

$$
\begin{aligned}
E[M_{T+1}] &= \sum_{i=1}^{T} R^{T-i+1}(\gamma-1)\left(\frac{X[T-i]AE[p^{\frac{1}{1-\beta}}]}{R^{T-i+1}\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\
&+ \quad X[T]AE[p^{\frac{1}{1-\beta}}]K_0 + R^{T+1}M_0,
\end{aligned}
$$

where

$$
X[j] = \sum_{k=0}^{j} R^k(1-\delta)^{j-k}.
$$

□

Lemma 3 (Two period p_0^*).

$$
p_0^* = \left(\frac{\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} - RM_0}{AK_0}\right)^{1-\beta}.
$$

Proof. Set $T-1$ cash flow (II.1) equal to $T-1$ target investment (II.5):

$$
AK_{T-1}p_{T-1}^{*\frac{1}{1-\beta}} + RM_{T-1} = \left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}}.
$$

Solve for p_{T-1}^* and set $T-1=0$. □

Proposition 2 (Two-period Firm). *Expected value for a 2-period firm is:*

$$
E_0[M_2] = E_0[M_2|U_0]Pr[U_0] + E_0[M_2|C_0]Pr[C_0]
$$

where $E_0[M_2|U_0]Pr[U_0] =$

$$(1-F[p_0^*]) \times \left(R(\gamma-1)\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} + (1-\delta+R)AE[p^{\frac{1}{1-\beta}}]K_0 + R^2M_0\right)$$

and $E_0[M_2|C_0]Pr[C_0] =$

$$\exp\left[F[p_0^*]\int_0^\infty \log\left(Ap_T^{\frac{1}{1-\beta}}\right)dp_1 + \int_0^{p_0^*}\log\left(\frac{1}{q}\left(AK_0p_0^{\frac{1}{1-\beta}}+RM_0\right)^{\frac{1}{\gamma}}+(1-\delta)K_0\right)dp_0\right]$$

Further, cash investment is:

$$\begin{aligned} E_0[I_0] &= (1-F[p_0^*])\left(\frac{AE[p^{\frac{1}{1-\beta}}]}{R\gamma q}\right)^{\frac{\gamma}{\gamma-1}} \\ &+ \exp\int_0^{p_0^*}\log\left(AK_0p_0^{\frac{1}{1-\beta}}+RM_0\right)dp_0 \\ E_0[I_1] &= 0. \end{aligned}$$

Proof. The constrained firm is like the unconstrained one, equation (II.6), except that investment may be constrained by cash flow. When $p_{T-1} < p_{T-1}^*$ investment is equal to cash flow:

$$Ap_{T-1}^{\frac{1}{1-\beta}}K_{T-1} + RM_{T-1} = ((K_T-(1-\delta)K_{T-1})q)^\gamma.$$

Solving for K_T:

$$K_T = \frac{1}{q}\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}}+RM_{T-1}\right)^{\frac{1}{\gamma}} + (1-\delta)K_{T-1}.$$

The cash equation for period T becomes $M_{T+1}|p_{T-1} < p_{T-1}^* =$

$$Ap_T^{\frac{1}{1-\beta}}\left(\frac{1}{q}\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}}+RM_{T-1}\right)^{\frac{1}{\gamma}}+(1-\delta)K_{T-1}\right), \quad \text{(II.10)}$$

This equation is a product of random realizations. Its expected value (see discussion) is $E[M_{T+1}|p_{T-1} < p^*_{T-1}] =$

$$\exp\left[\int_0^{p^*_{T-1}}\int_0^\infty \log A p_T^{\frac{1}{1-\beta}} \times \left(\frac{1}{q}\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}} + RM_{T-1}\right)^{\frac{1}{\gamma}} + (1-\delta)K_{T-1}\right) dp_{T-1}dp_T\right] \quad \text{(II.11)}$$

The double integral simplifies, so $E[M_{T+1}|p_{T-1} < p^*_{T-1}] =$

$$\exp\left[F[p^*_{T-1}]\int_0^\infty \log A p_T^{\frac{1}{1-\beta}} dp_T + \int_0^{p^*_{T-1}} \log\left(\frac{1}{q}\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}} + RM_{T-1}\right)^{\frac{1}{\gamma}} + (1-\delta)K_{T-1}\right) dp_{T-1}\right] \quad \text{(II.12)}$$

We already have a value for the unconstrained $p_{T-1} > p^*_{T-1}$ case, equation (II.7). Its arithmetic average (II.4) converges. For the constrained case (II.10) the geometric average (II.12) converges. The probability-weighted average of these, again, converges, being a linear combination of convergent sums. Set $T = 1$ and the result follows. □

Corollary 1. *Expected cash flow for a 2-period always-constrained firm is:*

$$E_0[M_2] = \frac{A^{\frac{\gamma+1}{\gamma}} K_0^{\frac{1}{\gamma}}}{q} \exp\left(\frac{1+\gamma}{\gamma(1-\beta)} E[\ln(p)]\right)$$

Cash investment is:

$$E_0[I_0] = AK_0 e^{\frac{1}{1-\beta}E[\log p]}$$
$$E_0[I_1] = 0$$

Proof. Let $F[P^*_{T-1}] = 1$, $M_{T-1} = 0$ and $\delta = 1$ then expected cash flow $E_{T-1}[M_{T+1}]$, (II.10) using (II.11) simplifies: $E_{T-1}[M_{T+1}]$

$$= \exp\left(\int_0^\infty\int_0^\infty \log\left(\frac{Ap_T^{\frac{1}{1-\beta}}}{q}\left(AK_{T-1}p_{T-1}^{\frac{1}{1-\beta}}\right)^{\frac{1}{\gamma}}\right) dp_T dp_{T-1}\right)$$

$$= \frac{A^{\frac{\gamma+1}{\gamma}} K_{T-1}^{\frac{1}{\gamma}}}{q} \exp\left(\frac{1}{1-\beta}\int_0^\infty \ln(p_T)dp_T + \frac{1}{\gamma(1-\beta)}\int_0^\infty \ln(p_{T-1})dp_{T-1}\right)$$
$$= \frac{A^{\frac{\gamma+1}{\gamma}} K_{T-1}^{\frac{1}{\gamma}}}{q} \exp\left(\frac{1+\gamma}{\gamma(1-\beta)}E[\ln(p)]\right).$$

Set $T = 1$ and the result follows. □

Proposition 3 (Additive Mean-preserving Spreads). *If an expected value is a linear combination of (1) positive random variables raised to a power greater than one, and (2) positive increasing functions of expectations of positive random variables raised to a power greater than one, and (3) constants, then the function is increasing in a mean-preserving spread of its random variables.*

Proof. Rothschild & Stiglitz (1970) show that a mean-preserving spread in a random variable representing a (non-negative) consumption good decreases expected utility for a risk averse agent—one with a concave utility function. By symmetry a mean-preserving spread increases expected value when the value function is convex. Let $g(p) = p^g, g > 1, p$, a positive random variable. Then $\frac{d^2 g(p)}{dp^2} = g(g-1)p^{g-2} > 0$ so $g(p)$ is increasing and convex and mean-preserving spreads in $\tilde{p}$ increase the value of the expectation $E[g(p)]$. Let $h(\cdot)$ be a positive increasing function. Then $h(E[g(p)])$ is increasing in mean-preserving spreads in $\tilde{p}$ since such spreads increase $E[g(p)]$. A linear combination plus a constant of such functions is again increasing in mean-preserving spreads in any of its random variables. □

Proposition 4 (Multiplicative Mean-preserving Spread). *If an expected value is a positive increasing product of random variables transformed by twice differentiable functions $g_i(\cdot)$ such that $g_i' g_i' > g_i'' g_i, \forall g_i$, then the expected value is decreasing in a mean-preserving spread of its random variables.*

Proof. Let $E[M]$ be an expectation of a positive increasing function of product of random variables transformed by a twice differentiable function:

$$E[M] \quad = \quad E[\prod_{i=1}^{I} h_i(g_i(p_i))],$$

where $h_i(\cdot)$ is positive and increasing and $g_i(\cdot)$ is a twice differentiable function.

$$\begin{aligned} E[M] &= E[\exp\left(\log k + \sum_{i=1}^{I} \log h_i + \log g_i(p_i)\right)] \\ &= \exp\left(\log k + \sum_{i=1}^{I} (\log h_i + E[\log g_i(p_i)])\right) \end{aligned}$$

This step follows if the expectation is to be convergent as the number of random variables increases without limit.

$$\frac{d^2 \log(g_i(p_i))}{dp_i^2} \quad = \quad \frac{g_i'' g_i - g_i' g_i'}{g_i^2}$$

If $g_i' g_i' > g_i'' g_i$ then $\log(g_i(p_i))$ is concave its expectation decreases with mean-preserving spreads in prices. The rest of the expression being deterministic, $E[M]$ also decreases with mean-preserving spreads in prices. □

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Chapter III

Ethics, Economics and Lawyers' Conflicts of Interest

ABSTRACT

When a lawyer represents more than one client, the effect of common agency on clients depends upon the nature of the strategic interaction between the clients. Common agency can be synergistic, destructive or neutral. A simple game theory approach to the relationship between the principals distinguishes these three situations and shows when common agency is relatively efficient from the clients' perspective. A reputational dynamic imperfectly implements efficiency. Third party enforcement through the institutions of legal discipline can encourage efficient behavior if the legal rules are efficient. I show that the U.S. law regarding lawyers common agency is usually efficient in that the outcomes of the cases are aligned with efficiency. Further, while the frameworks are very different, legal analysis and economic analysis tend to treat similar cases similarly. Where economics and law differ, the analysis may suggest how to make the positive law of lawyers' conflicts of interest more efficient.

III.A Introduction

When a lawyer represents more than one client, the effect of common agency on clients depends upon the nature of the strategic interaction between the clients. Common agency can be synergistic, destructive or neutral. A common agency is synergistic if all the principals may gain from the commonality; common agency is destructive if a principal necessarily loses; and common agency is neutral otherwise.

Dealing with common agency conflicts can be difficult. Neither the principals nor the agent seeks social welfare, or even a result that is Pareto optimal among the principals and agent[1]. Lawyers as agents seeking retainers have an incentive to preserve common agency irrespective of its value to principals; principals have insufficient information to assess conflicts; and third party enforcement is *ex post.* None of the actors has strong incentives and the information needed to avoid harms arising from destructive common agency while preserving synergistic common agency. Worse, the nature of the strategic interactions that arise in common agency are poorly understood.

The purpose of this paper is to develop a deeper understanding of the strategic interactions that arise in lawyers' common agency. A simple game theory perspective on the relationship between the principals distinguishes synergistic, destructive and neutral common agencies and shows when common agency is relatively efficient from the clients' perspective. With this central issue clarified, various exceptions and qualifications are easy extensions.

A self-enforcing reputational dynamic affecting lawyers imperfectly implements efficiency. Third party enforcement through the institutions of legal discipline also can encourage efficient behavior if the legal rules are effi-

[1]In this paper, "efficient" means Pareto optimal among the principals and agent. Social welfare is outside the scope of the discussion.

cient. I show that the U.S. law regarding lawyers' common agency is usually efficient in that the outcomes of the cases are aligned with efficiency. Further, while the frameworks are very different, legal analysis and economic analysis tend to treat similar cases similarly. Where economics and law differ, the analysis may suggest how to make the positive law of lawyers' conflicts of interest more efficient.

The explicit goal of this paper is to offer an economic model of how *ex ante* rational clients want their lawyers to deal with conflicts and how these results are reflected in law. Implicit larger goals are to give game theorists another institutional setting about which to think, and to give legal ethicists sharper tools with which to shape their canonical problems.

There are many ways to organize an analysis of lawyers' common agency. A legal scholar might start with the norms embodied in the statutory law, continue with their interpretation and application in particular cases, discuss exceptions or anomalies in the law, and conclude with the implications of the regularities and inconsistencies for subsequent cases. By contrast, an economist might make some normative and simplifying assumptions, develop an axiomatic model, continue by testing the correspondence between the model and empirical facts and conclude with a discussion of why and where the model and the facts differ. One who respects both methods, having unlimited time and space, would do both and profit from their synthesis.

Being constrained, I move quickly to comparisons between the economic and legal frameworks using examples from the case law. This brings us immediately to application of theory to facts. More leisurely development of economic and legal analyses are in appendices.

Here is a road map to what follows: In Section 2, I briefly outline the economic analysis and the legal institutions and analytical framework.

Appendix A does the economics more carefully and patiently; Appendix B does the same for the institutional setting and the law. The heart of the paper is three case studies in Section 3. *Aetna Cas. & Sur. Co. v. United States*, 570 F.2d 1197 (4th Cir 1978)(*Aetna*) illustrates synergistic common agency. In terms of game theory it is either a Prisoner's Dilemma or a Stag Hunt. *Fiandaca v Cunningham*, 827 F.2d 825 (1st Cir 1987)(*Fiandaca*) illustrates a destructive common agency. It is a Matching Pennies Game. Finally, *Universal City Studios, Inc. v. Reimerdes*, 98 F.Supp.2d 449 (SDNY 2000) (*Universal*) illustrates a neutral common agency. In the Discussion, Section 4, these three cases are supplemented by 14 more cases whose details are given in Appendix C. From the correspondence between efficient and empirical outcomes in these 17 cases, I argue inductively that the law is usually efficient. I also argue that the legal interests analysis is consistent with efficiency, with an important exception. A brief conclusion follows. The appendices are necessary to support the economic and legal arguments in the main text, and, collectively, are longer than the main text, but they may be set aside on a first reading.

III.B The Economic and Legal Frameworks

Before the economic and legal analyses can be applied, they must be described.

III.B.1 The Economic Framework

Three economic tools are most important: the Coasian analysis, the normal-form game, and the reputational dynamic.

According to Coase, the law does not matter for efficiency if transactions costs are negligible. If the allocation of legal rights is inefficient, the parties will renegotiate to an efficient outcome. Hence lawyers' conflicts of interest rules should be contractible.

There are many contractual tools to deal with agency problems, but they work poorly for lawyers' common agency. The analysis supporting this conclusion is given in Appendix A.1, but, summarily: Contactual tools and default rules fail in this situation due to asymmetric information and the lawyers' interest in specialization. Lawyers know more about lawyers' common agency problems than do their clients, and will, if rational, use that knowledge to distort the lawyer/client relationship to their own advantage. While lawyers' conflict of interest law limits freedom of contract between lawyer and client, such regulatory restrictions are justified under a modern view of the Coase Theorem because of market imperfections and asymmetric information (Macey & Miller (1997)).

Having shown that allocation of the common agency right is important because negotiation is hampered by asymmetric information, I turn to a game theory model. Since game theory is the study of strategic interactions, it is a natural for common agency issues. The most important issues underlying the conflicts of interest between principals can be adequately represented in 2 by 2 normal-form games [2], with a catalog of five types arising from variations in principals' payoffs. Notation and a more complete development of this

[2]The game theory in this paper is simple. The game is always a 2 by 2 normal-form game. This means there are two players (here principals); each chooses between one of two strategies, simultaneously without the knowledge of the other's choice. Outcomes depend upon their joint choices. A game can be represented in a table as in the top of Figure III.1. In that game, the players are the U.S. and its controllers. U.S. can choose u or d; controllers can choose l or r. If the pair of choices is (u, l), then U.S. receives a payoff of 1 and controllers receive 4. These numbers may represent the ranking of possible results in litigation. A 4 might be a complete win; a 1 is the worst possible outcome. In this figure, if U.S. guesses that controllers will play l, it can do better by playing d, in which case U.S. and the controllers both receive 2. The pair of strategies (called a "strategy profile") (d, l) yields the pair of outcomes $(2, 2)$. The (d, l) strategy profile, while Pareto inferior to strategy profile (u, r) and outcomes $(3, 3)$, constitutes a Nash equilibrium; neither U.S. nor controllers can do better without help from the other. What the strategies l, r, u, and d and the outcomes represent depends upon the case, and will be discussed primarily on a case-by-case basis; however, five game types characterized by the nature of their strategic interaction are developed in Appendix A.2. The type for the game in Figure III.1 is a Prisoners Dilemma θ_P.

model is given in Appendix A.2.

Three game types, Prisoner's Dilemmas θ_P, Battles of the Sexes θ_S and Stag Hunts θ_2, are synergistic in that common agency enables Pareto optimal cooperation or coordination. In (Perturbed) Matching Pennies θ_M, the principals' strategic interaction is purely competitive. A common agent necessarily favors one principal over another and so interferes with the agent's duty to act consistently with each principal's individual rationality constraint[3]. Because a common agent will break one principal's individual rationality constraint, common agency is destructive. Finally, in One Equilibrium Games θ_1, common agency is neutral in equilibrium for principals, but agents favor common agency because of their specialization interest. The agent's specialization interest is the agent's incentive to represent many principals with similar problems in order to develop expertise.

A norm emerges: An efficient institutional design allows conflicts in neutral cases and under certain conditions allows conflicts in the synergistic types, but an efficient design will prohibit conflicts in destructive types.

While the game theory model shows that conflicts of interest can be synergistic, destructive or neutral in the principals' game, there is no pressure that pushes agents toward efficient treatment of conflicts. If, however, a lawyer values her reputation, then there may be a simple dynamic which tends to drive agent behavior toward efficient outcomes. A highly stylized model of how this might happen is Appendix A.3.

These three economic tools, Coasian analysis, normal-form game theory and dynamics, will find application in the case studies to follow, but first I need to introduce the institutional side and the legal analysis.

[3]An unstated assumption here is that only pure strategies are allowed. Mixed strategies and randomization are considered in Appendix A.2.

III.B.2 The Legal Framework

While called "ethics" and "rules" or "canons," the law of legal discipline has teeth. It is explicitly stated as compulsory rules, not admonitions or exhortations, and the rules are enforced with vigor. The institutions of legal discipline are described in Appendix B.1.

A leading authority on legal ethics is the *Restatement Third, The Law Governing Lawyers*(*Restatement*), including "pocket parts" bringing it current. A *Restatement* represents the efforts of a committee legal scholars acting under the umbrella of the American Law Institute to clarify and synthesize the law, distilling it from the myriad reported cases, thereby providing guidance about how the law may apply in novel cases. There are *Restatements* on many legal subjects and they are more than academic exercises; judges cite them as authoritative expositions of the law.

Restatement §128 addresses lawyers' common agency in terms of four legal interests: loyalty, confidentiality, coordination and process integrity. Each client has the right to a lawyer's *loyalty*, a zealous fidelity to the client's interests. Each client also has the right to *confidentiality*, preservation of the client's confidences. On the other hand, common agency may aid *coordination* in which both clients gain relative to uncoordinated action. Finally, common agency may affect *process integrity*, the ability of the adversarial process to reach just decisions. The court may weigh these four interests on a case-by-case basis in deciding whether common agency has a synergistic, destructive or neutral role. (For more *see* Appendix B.2.)

Courts do not frequently make explicit use of the legal interests analysis. While the legal interests analysis arguably undergirds judical thinking and reconciles decisions, many cases have been justified with less articulate reasoning.

Tool kit in hand, I now turn to three case studies.

III.C Case Studies

In each of the following three case studies I describe a reported case, and then apply the economic and legal analyses developed in this paper. No one case has more than a few of the elements necessary to illustrate the entire scope of the analysis.

III.C.1 Aetna

Aetna Cas. & Sur. Co. v. United States, 570 F.2d 1197 (4th Cir 1978)(*Aetna*) illustrates a synergistic common agency. The following discussion begins with the facts and the statutory law, and continues with several different approaches to the case, starting with those of the parties and judges.

Facts: Aetna paid $25,000,000 in claims arising from a plane crash, and then sued the United States and four of its air traffic controllers for negligence in connection with the crash. Aetna objected when the Department of Justice represented both the United States and the controllers, arguing that the United States on one hand and the controllers collectively on the other hand had differing interests and so required separate representation. The lawyer for the United States argued that there was no conflict. The controllers appeared in court, along with counsel for their union. They did not merely consent, they demanded common agency with the United States. Nevertheless, the trial court found a fatal conflict of interest, and ruled that the defendants required separate representation; the Court of Appeals reversed.

Law: Some of the North Carolina law applicable in this case, DR5-105, based on the ABA Code and comparable to the ABA Rules (Appendix B.2.) was:

(A) A lawyer should decline proffered employment if the exercise of his independent professional judgement in behalf of a client will be or is likely to be adversely affected by the acceptance of the proffered employment, except to the extent permitted under DR5-105(C).

(B) A lawyer shall not continue multiple employment if the exercise of independent professional judgment in behalf of a client will be or is likely to be adversely affected by his representation of another client, except to the extent permitted under DR5-105(C).

(C) [A] lawyer may represent multiple clients if it is obvious that he can adequately represent the interests of each and if each consents to the representation after full disclosure of the possible effect of such representation on the exercise of his independent professional judgment on behalf of each.

Trial Court: The trial court pointed out contentions each of the four controllers might make to exculpate himself from liability, but cast blame on others. As counsel for Aetna put it: "[W]hen the government attorney represents everybody in a piece of litigation [so] that the position taken by everybody is uniform, * * * there is almost a conspiracy of silence as to what truly happened * * * ." The trial court found that the controllers' consent could "not be presumed to be fully informed when procured without the advice of a lawyer that has no conflict of interest." (For an example in which the trial court's argument is compelling, *see, Dunton v. County of Suffolk,* 729 F.2d 903 (2nd Cir 1984) (*Dunton*)[4].) The trial court ruled that it was not "obvious" that the Attorney General could adequately represent the interests of each defendant. There was an "actual" conflict and it was unacceptable.

Appellate Court: The appellate court found no "actual" conflict, only the trial court's hypotheticals and Aetna's hopes. The purpose of the rule was to protect the defendants in this case, and they had freely chosen common agency. Indeed, the controllers gained access to the resources and expertise of the United States through common agency. It was "obvious" that the United States could adequately represent the interests of all the defendants.

Legal Interests Analysis: That the court focused on whether something should be labelled "actual" or "obvious" was an unfortunate outcome

[4]Descriptions of this and most of the other cases cited in the main text are given in Appendix C.

of the wording of statutes. Instead, one might think of the case in terms of the legal interests analysis. Two interests, defendants' confidentiality and defendants' loyalty interests, Aetna was not in a position to raise because those interests did not exist for its benefit. A third interest, coordination, cut against its claim. Aetna's strongest argument is about process integrity: Regardless of how the defendants benefit, Aetna argues that the adversarial system will not function properly in the presence of the defendants' conspiracy of silence. The process integrity argument can win, *Sapienza v. New York News, Inc.*, 481 F.Supp. 676 (SDNY 1979)(*Sapienza*). However, if parties aligned on the same side (e.g. as defendants) see a benefit from coordination, this outweighs the process integrity interest. This argument is no more satisfying than the Court's because it does not clearly articulate why or when coordination trumps process integrity.

Economics: Coasian Argument: The focal question is whether the controllers had sufficient unbiased information to make a decision on common agency. The trial court's position seems to be that each of the four controllers needed a lawyer of his own to support an informed decision not to have a lawyer of his own in the primary case. The Court of Appeals seems to have thought common agency through union counsel was sufficient, perhaps focusing on the potential conflicts between the U.S. and the controllers collectively. Coase does not help us assess whether transactions costs were "low" with union counsel or whether more decision support was necessary to render costs low.

Economics: Normal-Form Game Theory Argument: If we accept the Court of Appeals' position that the troubling conflict was between the U.S. and the controllers collectively, the next problem is to assess the structure of that conflict.

One might think of *Aetna* as a Prisoner's Dilemma θ_P, *see* Figure III.1. Think of the U.S. as the Row player and the controllers jointly as the Column player. Concretely (and entirely hypothetically), suppose d and r correspond to a minimalist defense, relying upon the idea that it may be difficult for Aetna to prove its case if the defendants are silent. On the other hand, u and l might correspond to putting on evidence that tends to confirm liability but place liability elsewhere. To be more concrete, the controllers' choice of r might correspond to pointing out equipment maintenance problems or errors of other controllers, while the U.S.'s choice d might correspond to arguing that the controllers' conduct was so far from what they were authorized to do, and so well concealed from superiors, that they were acting outside the scope of their employment. Perhaps, for example, they sought to cause a crash. While these arguments may not have been relevant in *Aetna*, others with similar strategic implications, as the trial court pointed out, may have been relevant.

The payoffs are shown on the Figure III.1 playoff graph. Feasible strategies, including mixed strategies, are the grey area. Points to the right are best for the Controllers - Column; points high on the vertical axis are best for U.S. - Row. If the U.S. - Row plays u and the Controllers - Column play r then each get payoffs of 3; perhaps this equates to Aetna losing the case entirely; however, the Pareto optimal pure strategy profile (u, r) is not an equilibrium in separate agency. If the U.S. - Row plays u, Controllers - Column's best response is l, perhaps casting blame on equipment or operating procedures, and if Controllers - Column plays r, U.S. - Row's best response is d, perhaps arguing that the controllers were acting outside the scope of their employment or with malice. If either the U.S. or controllers deviate from (u, r) they break the "conspiracy of silence" and help Aetna; and it may be rational for both to deviate. The defendants' Nash equilibrium is a (d, l), a Pareto inferior strategy profile.

The principals may be able to avoid this inferior outcome if a common agent can impose the *ex ante* Pareto superior (u, r) strategy profile. This is the common agent arbitrator outcome set A^*[5].

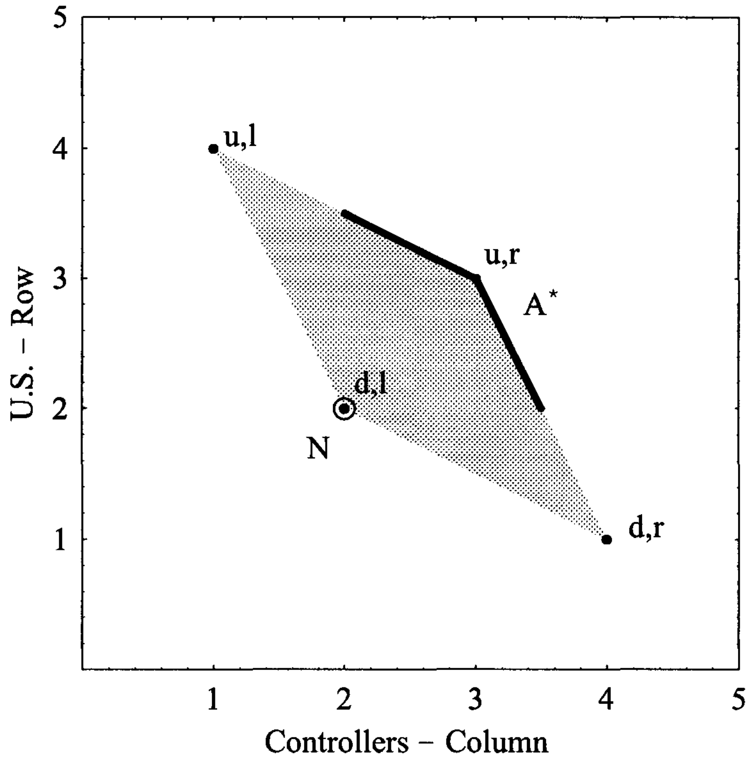
[5]All of the solution sets N, cN, A, A^*, M, M^* and X in the graphs are defined in Appendix A.2. These concepts can be ignored on a first reading.

Figure III.1: Prisoner's Dilemma θ_P

U.S.\Controllers	l	r
u	4 1	3 3
d	2 2	1 4

Payoff Space

N—pure Nash non-cooperative equilibrium (a point)
M^*—mediator equilibrium (empty)
A^*—arbitrator solution (kinked line)



There are other game theory types that might fit *Aetna.* The United States and controllers are asymmetric. The United States is the deep pocket; the controllers know certain facts of the case best. In the Stag Hunt θ_2, Figure III.2 the idea is that there are two equilibrium strategy profiles, but both parties prefer one equilibrium to the other. The two equilibria in *Aetna* might be the best result under separate agency, and the best equilibrium under common agency. The inferior equilibrium is (u, r), yielding each principal 3; the superior equilibrium is (d, l), yielding each principal 4. I have classified *Aetna* as a Stag Hunt θ_2, but reasonable people might differ; on the other hand, it seems clear that the strategic interaction is characterized by possible synergies.

Whether *Aetna* is a Prisoner's Dilemma θ_P or a Stag Hunt θ_2, common agency is efficient. There is an important difference between the two games. In the Prisoner's Dilemma θ_P, the players have an incentive to defect *ex post.* Coordination requires some form of enforcement. I call this enforcement "arbitration". In the Stag Hunt θ_2, if both parties find themselves playing the inferior equilibrium, neither will defect, but if a mediator suggested the superior equilibrium they have no reason not to choose it. The Prisoner's Dilemma θ_P is an example in which common agent arbitrators can obtain cooperation when common agent mediators fail. In the Stag Hunt θ_2, either a mediator or an arbitrator can secure coordination.

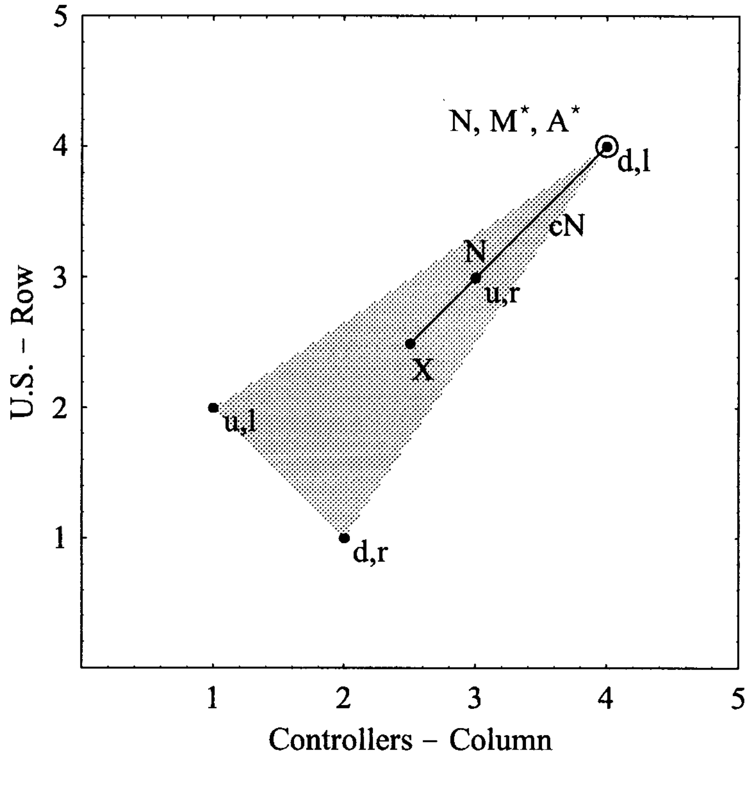
Informed consent is another key element in the analysis of *Aetna.* While game theory does not tell us whether there was enough information, it does supply insight on why it is important that principals be informed. The controllers and U.S. are in a long-term employment relationship of which this case is only one stage, albeit presumably an intense stage. The controllers and U.S. are playing a repeated game. Analysis of repeated games is essentially different from stage games like the Stag Hunt θ_2 because, pursuant to the "folk theorem," there is typically a very large set of possible equilibria and choice among them depends on players' histories, expectations, long-term strategies

Figure III.2: Stag Hunt θ_2

U.S.\Controllers	l	r
u	1, 2	3, 3
d	4, 4	2, 1

Payoff Space

N—pure Nash equilibria (2 points)
X–mixed Nash equilibrium
cN—convex hull of Nash equilibria (line from X to (d, l))
M^*—mediator equilibrium (a point)
A^*—arbitrator equilibrium (a point)



and preferences. Neither a common agent nor a third-party enforcer is likely to be knowledgable about the repeated game issues between the principals. Accordingly, it is difficult or impossible for an agent or court to decide for the principals, given that the decision is payoff relevant in the repeated game. Hence, the principals must be educated as to the tradeoffs and informed consent obtained when the common agency has the potential to be a synergistic stage in a repeated game.

Economics: Dynamics and Reputation: There may also be a repeated game played by the agent in which the agent's reputation is the crux. Since all of the time of the common agency lawyer in *Aetna* is committed to the U.S., reputational dynamics is at most a remote and indirect factor in this particular case.

In Sum: Coordination among the U.S. and controllers may do nothing to further social welfare, but the focus of the law regarding the lawyer's role is appropriately on what best serves the lawyer's clients. Aetna's and the social interest in delving into the details of the case should be protected by the law of discovery. Aetna is making explicit recourse to third-party enforcement to prevent Pareto optimal (as between the U.S., controllers, and their lawyers) coordination that does not further its interest. The Court of Appeals refused to intervene. *Aetna* is a synergistic Stag Hunt θ_2 between the U.S. and the controllers. Allowing them to coordinate on the best equilibrium benefits both. This case illustrates several other features of what follows: (1) The cases are full of important detail, sometimes interesting, sometimes annoying. (2) Reasonable people might disagree about which of the game types applies; judgment has been exercised. (3) In reviewing *Aetna* I described seven perspectives (plaintiff's, defendant's, trial court's, appellate court's, legal analysis, Coasian analysis and the normal-form game theory argument) on the conflicts of interest. While this paper argues that the normal-form game theory model is superior on balance because it obtains sharp and efficient results, there are many ways of thinking about the conflicts of interest in most of the cases.

III.C.2 Fiandaca

Fiandaca v Cunningham, 827 F.2d 825 (1st Cir 1987)(*Fiandaca*) illustrates a destructive common agency, one in which the relationship between the principals is so conflicted that the agent cannot act consistently with the interests of both.

Facts: Women prison inmates ("Prisoners") sued New Hampshire claiming that prison conditions for them were worse for them than for male prisoners, in violation of constitutional law. In an unrelated action, a group of mentally retarded residents of New Hampshire's Laconia State School ("LSS"), the "Garrity" class, sued the State over conditions at LSS. New Hampshire Legal Assistance ("NHLA") along with others, represented both the Prisoners and Garrity. Prisoners and Garrity both won.

New Hampshire proposed to implement the Prisoner's award by moving them to LSS pending construction of a new prison, displacing some Garrity residents. Naturally, the Garrity residents opposed this proposal. NHLA rejected the LSS proposal on the grounds it would harm Garrity. New Hampshire moved to disqualify NHLA as counsel for Prisoners due to NHLA's conflict of interest. The trial court ruled that New Hampshire's proposal to use LSS was unacceptable for *Prisoners* and so any conflict in NHLA's representation of both Garrity and the Prisoners was irrelevant. The appellate court reversed.

Law: The applicable conflicts law was New Hampshire's, which had adopted a variation on the ABA Model Rules, specifically Rule 1.7(b)[6]. A relevant portion of the New Hampshire Rules of Professional Conduct stated

> A lawyer shall not represent a client if the representation of that client may be materially limited by the lawyer's responsibilities to another client ... unless:
>
> (1) the lawyer reasonably believes the representation will not be adversely affected; and
>
> (2) the client consents after consultation and with knowledge of the consequences.

On appeal the court also relied on ABA commentary concerning Rule 1.7(b):

[6] The Model Rule is discussed in Appendix B.2

> Loyalty to a client is also impaired when a lawyer cannot consider, recommend or carry out an appropriate course of action for the client because of the lawyer's other responsibilities or interests. The conflict forecloses alternatives that would otherwise be available to the client.

Trial Court: The trial court noted that the litigation had dragged on expensively for years. Disqualification of NHLA would run up costs, and further delay relief for Prisoners. The court noted that it would not accept the proposal to transfer Prisoners to LSS anyway and ordered New Hampshire to find another alternative. In effect the judge decided that he would protect Garrity in the Prisoner's litigation, so Garrity did not require independent representation.

Appellate Court: The LSS proposal may have been a good one for the Prisoners, but NHLA was foreclosed from viewing it favorably because of its affect on NHLA's other client, Garrity. The conflict is direct. The Court ruled "[W]e are unable to identify a reasoned basis for the district court's [decision;] we hold that its order amounts to an abuse of discretion."

Legal Interests Analysis: In *Fiandaca* common agency violates the loyalty and the confidentiality interests. It violates loyalty because the LSS proposal may be a good one for Prisonsers and bad one for Garrity. NHLA's position on the proposal will violate the interests of one of its clients. It also violates the confidentiality interest. NHLA may have learned information from the Garrity class that it may want to use in advocacy of the proposal and it may have learned information from the Prisoners that it may want to use in advocacy against the proposal. There is a coordination interest, here advocated by the trial judge who wishes to avoid additional delay in the Prisoner's case, but it is coordination between the State and the Prisoners, not the Prisoners and Garrity. Finally, the process integrity interest is involved. Garrity sought to intervene in the Prisoner's case, a formal procedure allowing it to advocate its interests. It is unclear who represented Garrity in the motion to intervene, but it was denied in the judge's attempt to minimize NHLA's problems. Since Garrity clearly had a stake in the Prisoner's settlement, this violated process integrity. From the perspective of the relevant principals, Prisoners and Garrity, the weighing process is easy here; the conflict is not allowable.

Economics: Coasian Argument: It is doubtful whether any of the Plaintiffs have independent and voluntary decision making capacity, being mentally incapacitated or incarcerated. But the real problem is timing. This conflict did not arise until after NHLA had won on the liability issue in both cases, and there is no evidence that any Plaintiffs or their lawyers anticipated the conflict of interest. One cannot rely on an *ex ante* contractual solution to the unforeseeable *ex post* conflict between Garrity and Prisoners.

Economics: Normal-Form Game Theory Argument: Fiandaca is easy to classify as Perturbed Matching Pennies, Figure III.3. Let u and r represent acceptance of the proposal. No matter what profile is chosen, one of the principals would do better with a different strategy.

Suppose NHLA proposes outright acceptance, (u, r). Prisoners gain a payoff of 4 from this profile and Garrity get 1. But knowing Prisoners' strategy, Garrity's best response is l, opposition to the settlement. NHLA's Failure to play l in response to u fails to be consistent with zealous fidelity to Garrity's individual rationality constraint. But if NHLA chooses l with Garrity in mind, Prisoners' best response is d, perhaps agreeing quickly that the proposal is unacceptable in order to promptly develop alternatives, and failure to advocate (d, l) fails to be consistent with loyalty to Prisoners. The process continues; (d, l) is disloyal to Garrity and the Garrity best response yields (d, r), which in turn yields (u, r), where we started. As the payoff graph illustrates, whatever helps Row hurts Column and vice versa. There is no equilibrium in pure strategies in this game.

Since there is an equilibrium in mixed strategies, a theorist might suggest NHLA toss a coin to choose a strategy profile to recommend to its clients, but implementation of this mixed strategy would be surreal. It is implausible to suggest that Prisoners and Garrity could and would make a binding agreement toss a coin, but even if they agree and were bound to such a proposal, the choice is more complex than a coin toss. It is likely that there are many possible responses building on the State's proposal and it is NHLA's job to develop them (perhaps in negotiation with the State) in a direction favorable to the client, assess them for the client and then advocate the client's choice. It cannot engage in this process faithfully to both clients at once. However one interprets mixed strategies, they do not resolve the conflict.

Economics: Dynamics and Reputation. As in *Aetna*, common agency counsel in this case is not in private practice and the dynamic mechanism is arguably irrelevant.

Conclusion. Disqualification in a Matching Pennies Game θ_M is efficient as between Garrity and Prisoners, and disqualification is the result obtained by third party enforcement in this case.

III.C.3 Universal

Though the underlying litigation was far from trivial to the parties, *Universal City Studios, Inc. v. Reimerdes*, 98 F.Supp.2d 449 (SDNY 2000) (*Universal*) illustrates a neutral common agency, a One-Equilibrium Game θ_1.

Facts: J.K. Rowling, Scholastic, Inc. and Time Warner owned the rights to the *Harry Potter* books. This group was engaged in a series of legal actions to protect their rights to Harry. When Stouffer claimed the copyright and trademark rights to the term "muggles", it was Scholastic's turn to handle the defense at its expense through its counsel, the Frankfurt firm. Though Time Warner was therefore a client of the Frankfurt firm with all the duties entailed, there was little contact beyond providing copies of legal papers.

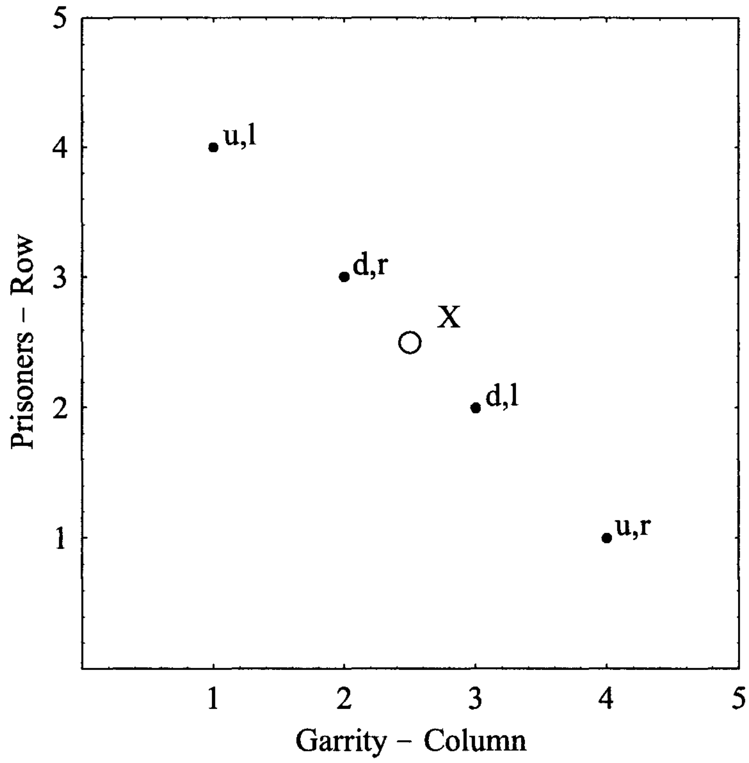
In another time and place Time Warner and the other "major motion picture studios" sued Corley and others for releasing on the web software to defeat DVD encryption. During the course of the suit, the Frankfurt firm became counsel to defendant Corley. Time Warner objected. Frankfurt offered to withdraw from representation of Time Warner in the Harry Potter case;

Figure III.3: Matching Pennies θ_M

Prisoners\Garrity	l	r
u	4 1	1 4
d	2 3	3 2

Payoff Space

N—pure Nash equilibrium (empty)
X—mixed Nash equilibrium (a point)
M^*—mediator equilibrium (empty)
A^*—arbitrator solution (empty)



Time Warner wanted Frankfurt off the DVD case. The trial court denied Time Warner's objection to Frankfurt's representation of Corley, but suggested that a separate disciplinary proceeding against Frankfurt might be appropriate.

Law: New York State law governed this case. New York had adopted the ABA Code, similar to that of North Carolina and *Aetna.* In both jurisdictions an explicit rule addressed "suing the client" by negative implication:

> "[A] lawyer may represent a client if ... the representation does not involve the assertion of a claim by one client against another represented by the lawyer in the same litigation or other proceeding before a tribunal ..." (Model Rule 1.7(b)(3).)

This rule appears straightforward; however, there are New York cases softening the rule, *Glueck v. Jonathan Logan, Inc.*, 653 F.2d 745 (2d Cir. 1981) (*Glueck*); *Commercial Union Insurance Co. v. Marco International Corp.*, 75 F. Supp. 2d 108 (SDNY 1999) (*Commercial Union*). In *Glueck* a lawyer sued a member of an association on a matter unrelated to his representation of the association. In *Commercial Union* a lawyer represented an insurance company against an insured in litigation over coverage, while representing the insured as nominal plaintiff in another claim the insurer had paid and was pursuing under subrogation.

Trial Court: After considering various arguments, the court relied primarily on the lawyer's duty of loyalty to his client, and held that a violation of the ethics rules had occurred. However, the court also expressed concern that Time Warner's motion was motivated to inconvenience Corley, rather than any compromise of Time Warner's lawyer/client relationship, and refused to disqualify Frankfurt, saying "[t]he proper place for this controversy is in the appropriate professional disciplinary body."

Legal Interests Analysis: While Frankfurt's conduct violated the loyalty interest, it did not violate any confidentiality interest. Frankfurt's representation of Time Warner in the "muggles" case was completely unrelated to the DVD case and nothing of Time Warner's internal operating procedure, inclinations or habits were shared, unlike *In re Dresser Ind.*, 972 F.2d 540

(1992) (*Dresser*). The coordination interest is not implicated. If Corley was concerned that Frankfurt would not vigorously represent him because of Frankfurt's involvement with Time Warner, the process integrity interest would have been at stake, but the court noted that, if anything, Frankfurt was unusually zealous on Corley's behalf in the DVD case. In short, only the loyalty interest is at stake in a legal interests analysis of *Universal*. Note however, that the final result is to allow the conflict. The loyalty interest did not prevail.

Economics: Coasian Argument: As with *Fiandaca*, transactions costs were prohibitive in *Universal*. The Court noted that Frankfurt took on the DVD case without realizing that it had a conflict. Surely, if Frankfurt does even not know it is a common agent, it is impractical to expect the parties to negotiate *ex ante* over prospective conflicts in the relationship.

Economics: Normal-Form Game Theory Argument: In *Universal*, Frankfurt could and did compartmentalize its representations. Different lawyers handled the two representations and neither was even aware of the work of the other at first. The best strategy for Corley was completely unrelated to the best strategy for J.K. Rowling, Scholastic and Time Warner. There was no opportunity for coordination, but equally none for competition. Consider the One Equilibrium Game θ_1 given in Figure III.4. Suppose Corley is the Row player and Time Warner is the Column player. The unique equilibrium strategy profile is (d, l), where d is Time Warner's optimal strategy in the Harry Potter case and l is optimal for Corley in the DVD case. Time Warner would gain from Corley playing u, perhaps conceding defeat in the DVD case, but it would be irrational for Corley to play u.

The court noted a possible tactical gain to Time Warner from disqualifying Frankfurt from the DVD case just before trial (a "hold up" problem or "rent seeking") and Time Warner's delay in raising the issue, which arguably further disadvantaged Corley. However, aside from this possible tactical point, neither principal gains or loses from common agency. By contrast, the agent has a specialization interest in common agency (*see* Appendix A.1). Unlike

Aetna, in which client consent to the common agency is important because of the possible dynamics between the principals, there is no expected efficiency gain from requiring the principals' consent to the common agency in *Universal.* If Time/Warner - Column in Figure III.4 protests not getting a "4" when it will never be rational for Corley - Row to act in a way that makes it possible, Time Warner's complaint about an agent who does not produce a "4" is not justified. Indeed, while Time Warner wanted Frankfurt disqualified and Corley disadvantaged, it did not seek to dictate Corley's strategy.

Since Time Warner's consent could have been (and was) withheld without good cause, it should not be required. Even if the principals are playing a dynamic game, the outcome of litigation over common agency in a neutral stage game is not payoff relevant to the principals. Their consent is unnecessary for efficiency.

Economics: Dynamics: In Appendix A.3. I argue that behaving ethically enhances a lawyer's reputation and thereby her future employment. Acting unethically may have short-term benefits. When the case is large, the lawyer may have more of a temptation to act unethically. The DVD case was described by the trial court as a major case, and Frankfurt's behavior was labelled "unethical." Frankfurt may have suffered harm to its reputation by virtue of its failure to be aware of the conflict, and its failure to resolve the conflict once it knew of it. One may suspect that its conflicts checking procedures were revisited after this litigation. To the extent this happened, it was an inefficient punishment because Frankfurt's conduct was efficient despite violation of the loyalty interest.

Conclusion: The court reached the efficient result in this case, allowing common agency in a case in which it did no harm, even though common agency violated the letter of the law and the legal interests analysis. *Universal* is an example of the power of the economic analysis and the ability of judges to reach efficient results even when the law and the legal interests analysis are obstacles.

Figure III.4: *Universal* θ_1

Corley\Time Warner	l	r
u	1, 4	1, 3
d	4, 2	4, 1

Payoff Space

N—pure Nash non-cooperative equilibrium (a point)
M^*—mediator equilibrium (empty)
A^*—arbitrator solution (empty)

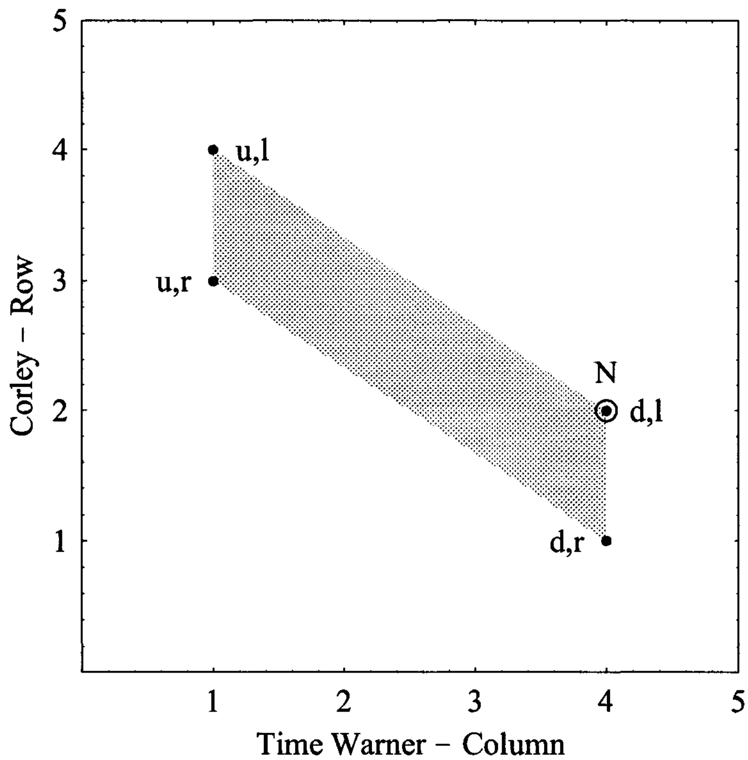


Table III.1: Game Types

Strategic Interaction	Pure Equilibria 0	1	2
Synergistic	na	θ_P	θ_S, θ_2
Neutral	na	θ_1	na
Destructive	θ_M	na	na

III.D Discussion

The three case studies are samples from a set of types of games and the corpus of reported cases. They illustrate the claim that reported cases can be thought of in terms of game types; that efficiency depends upon allowing common agency in some types but not others; and that the outcomes of the cases are usually consistent with efficiency. This section extends consideration to game types generally and the reported case law generally and shows that the law of legal ethics as interpreted by the courts is largely consistent with the game theory model exemplified in Section 3. Therefore, legal ethics reinforces efficient outcomes when reputational incentives are insufficient.

The games types, Prisoner's Dilemma θ_P, Battle of the Sexes θ_S, Stag Hunt θ_2, One Equilibrium Game θ_1 and Matching Pennies θ_M are summarized in Table III.1, and developed in much more detail in Appendix A.2.

In addition to *Aetna, Fiandaca* and *Universal*, in Appendix C I analyze many of the cases on common agency referenced in the *Restatement.* The omitted cases are redundant or outside the scope of the economic argument. *Sapienza* exemplifies the sort of case that might have been omitted as outside the scope. Each case is cataloged in terms of the game theory model and its type derived. Like most real data, the cases are messy, interesting for reasons beyond the present purposes, and classification is not always obvious.

Table III.2 summarizes the results. The cases are scattered among the types. In the 17 cases, the outcome was efficient in 15, uncertain in one, *MGIC Ind. Corp. v. Weisman*, 803 F.2d 500 (9th Cir. 1986)(*MGIC*), and inefficient

Table III.2: Catalog of Cases

Type	Number	Efficient	Consistent	Short Name
Synergistic				
θ_S, Battle of the Sexes	5	4	4	*Dunton*
				Hayes
				Hurt
				Ishmael
				Levine
θ_2, Stag Hunt	2	2	2	*Aetna*
				Klemm
θ_P, Prisoner's Dilemma	2	2	2	*Kerry*
				Messing
Neutral				
θ_1, One Equilibrium	3	3	2	*Spienza*
				Wait
				Universal
Destructive				
θ_M, Matching Pennies	5	4	4	*Dresser*
				Fiandaca
				Houston
				MGIC
				Worldspan
Total	17	15	14	

in one, *Hayes v. Eagle-Picher Ind. Inc.*, 513 F.2d 892 (10th Cir 1975)(*Hayes*). As with most attempts to assess case law empirically, this attempt is burdened by absence of a random sample. We see only cases litigated all the way to a reported opinion. We also have no ready null hypothesis about outcomes in a hypothetical random sample. Without a random sample or an alternative hypothesis it is not meaningful to ask whether the game theory model is a statistically significant predictor of outcomes. By contrast, I claim 15 out of 17 is to be a good rate of substantive success. By comparison, the outcomes of the cases are not always aligned with analysis of the *Restatement* either; *Universal*, *MGIC* and *Hayes* are arguably inconsistent. The game theory analysis offered here does better than a traditional legal interests analysis while constrained by much more structure.

The outcomes are efficient, but interpreting the law as nothing more than box scores may be too reductionist. Surely, one must also consider how the outcomes are reached. The legal interests analysis with its balancing of four values (loyalty, confidentiality, coordination and process integrity) is an appropriate framework for how legal scholars and judges would resolve conflicts questions.

There are close ties between three of the four values and the economic analysis: First consider the loyalty and confidentiality interests. These interests parallel the principal's individual rationality constraint. The loyalty and confidentiality interests overlap and I argue that only the confidentiality interest should be considered. The argument turns on the difference between One Equilibrium games θ_1 and Matching Pennies games θ_M. In the *Universal* One Equilibrium game θ_1, Figure III.4, the loyalty interest is violated when an efficient agent chooses an outcome that is not the favorite of one of the principals, but violation of that interest is efficient because the result is the unique Nash equilibrium outcome. In Matching Pennies θ_M both the loyalty and confidentiality interests are implicated. Not only does one principal lose, but the agent knows that principal will lose and has the information necessary to produce a better outcome for that principal. Confidentiality matters in Matching Pennies θ_M because the opponent's strategy is not knowable *ex ante*; in One Equilibrium games θ_1 the opponent's rational choice is knowable *ex ante*. Violation of the confidentiality interest can cause real and avoidable (given the agent's information) harm, but violation of the loyalty interest alone does not.

The loyalty interest has a strong emotional pull for courts and litigants, but is irrelevant for efficiency. On the other hand, violations of the confidentiality interest are important because they change outcomes. Whether the objection to a conflict is sustained depends upon whether the conflicting representations are overlapping sufficiently to make a Matching Pennies game θ_M. When the confidentiality interest is implicated (*Dresser* and *Worldspan, L.P. v. The Sabre Group Holdings, Inc.*, 5 F.Supp.2d 1356 (ND Ga 1998) (*Worldspan*)), they overlap too much. When only the loyalty interest is implicated, then, dicta and the letter of the law aside, the courts may not be very concerned (*Universal*).

A third legal value "coordination" is the label given to the potential gains from common agency. Coordination may be as simple as scale economies: two clients with the same (or similar) project can economize by having the same agent do the work. However, coordination through mediation or arbitration can have the more fundamental purpose highlighted by the synergistic games. In Battles of the Sexes θ_S and Stag Hunts θ_2, a common agent acting as a mediator can facilitate choice among multiple equilibria and thereby add value. In the Prisoner's Dilemma θ_P, mediation fails, but a common agent acting as an arbitrator can sustain a Pareto superior disequilibrium solution. In Aumann's Game, Figure III.7, a mediator helps, but an arbitrator is better.

The fourth legal value is "process integrity". This value is outside the scope of economic analysis, but arises in several cases. In *Hurt v. Superior Court,* 601 P.2d. 1329 (Ariz. 1979) (*Hurt*) the court is concerned that the interests of an infant be represented. In *Messing v. FDI, Inc.*, 439 F.Supp.

776 (1977) (*Messing*) it is a corporate entity with rights and responsibilities, but no will of its own. In *Sapienza* it is whether a dispute is real, or a show put on for the court. In *Hayes* its about decision-making within a group of aligned principals.

While the foregoing legal values analysis is very different from game theory, the confidentiality and coordination values have clear ties to the game theory model. Like the outcomes of the cases, the legal analysis tends to reach efficient conclusions. The economic model, reputational incentives and ethics law are mutually consistent and mutually reinforcing[7].

III.E Conclusion

In this paper simple economic tools are applied to an unusual problem. Legal ethics is not usually considered as an object for economic analysis.

The fundamental premises of legal ethics are those of every-day pragmatists. There is a body of law, detailed, generally accepted, and articulate, addressing the ethical behavior of lawyers facing conflicts of interest and many reported cases illustrating its enforcement. These cases supply a rich set of empirical facts and positive rules for the economist's consideration.

While economics is also pragmatic, ethics and economics start from opposite poles. Economists study the consequences of rational selfishness; ethicists study duties and selflessness. Economists value efficiency. Ethicists consider many values. Economists design for efficiency by mechanisms that, like Adam Smith's invisible hand, exploit selfishness for the common good. The world view of economics glibly eviscerates ethics by equating duty with

[7]These conclusions suggest a parenthetical point: Legal ethics are a part of a greater ethics of capitalism, a set of maxims to bridge failures of the many assumptions necessary for the theorems of welfare economics. To some extent, people behave ethically because they have been indoctrinated. They want to be, and to be seen as, people who behave ethically. There is evidence that these maxims work. For example, it is well understood that nations prosper with the rule of law and high levels of trust among citizens and suffer with corruption, hold-ups and rent-seeking. Efficient treatment of common agency issues is implemented through reputational incentives and law, but also though this indirect channel of influence on behavior.

self-interest and making efficiency the only value to be seriously considered.

Nevertheless, professional ethics is a fit subject for a rational choice model. Asymmetric information is the very stuff of professional practice and when information is incomplete or asymmetric, the purest forms of economic efficiency are rarely implementable. Yet the model developed in this paper is very simple as economics, and attuned the problem at hand.

This analysis has the potential to enrich legal analyses of conflicts of interest. More than pointing out isolated errors, like *Hayes* and the loyalty interest, game theory works as a model for conflicts; it sharpens and deepens thinking about the issues.

The game theory approach may generalize beyond the legal ethics of common agency. Corporate governance, public accounting, public officials, real estate agencies and medicine all are rife with common agency problems. Those problems tend to differ from the common agency issues in legal ethics in two key respects: (1) the law is typically less developed and demands less of agents, and (2) the principals in legal ethics quandaries are essentially symmetric; in other kinds of common agency there may be systematic asymmetries between the principals.

III.F Appendix: Economics

III.F.1 Ethics as Constraints on Freedom of Contract

Lawyers' common agency law restrains freedom of contract between lawyers and clients. In this appendix, I argue that the Coasian argument in favor of freedom of contract is weak in this instance. Laws restricting lawyers' common agency are justified because of market imperfections arising from asymmetric information.

The costs of getting the parties together, doing their homework and agreeing on terms, plus the expected costs of bargaining impasses all come under the rubric "transactions costs." The label is unfortunate. Transactions costs are all the whole vague heap of frictions and imperfections in coming to agreement, of which the costs of the documenting and communicating assent to an agreement may be among the less important. The Coase Theorem is the simple claim that in the absence of transactions costs the allocation of rights does not matter for efficiency (Coase (1960)). However, given their broad redefinition, transactions costs are almost always important, and so the allocation of rights almost always *does* matter. I will argue that lawyers' common agency rights are typical in this respect.

A principal may want exclusivity. An agent may wish to represent more than one principal. In the absence of a legal rule, the principals and agent may negotiate this point. According to Coase, if the law allocates the right in an inefficient manner, the parties modify the default rule. If the law allocates the right to a principal, but common agency is optimal, the parties will negotiate to vary the default rule to that end; on the other hand, if the law allocates the right to the agent, but exclusivity is more valuable to principals than common agency is valuable to the agent, again they will negotiate and re-allocate the right.

If transactions costs are positive, then it is efficient to allocate the right where negotiation would most often place it and minimize expected costs. If there is a pattern, say common agency is most often efficient in tort cases and exclusivity most often efficient in contract cases, then a more articulated rule taking advantage of the pattern is efficient. However, these are second-best results. When errors are inevitable because circumstances vary and the rule must be made *ex ante*, some frictional costs are inevitable and the result is inefficient (Farrell (1987)).

Choice of the remedy for breach is another institutional design element. One way to enforce a rule is by injunction. That is: The right is enforceable by an order that the parties abide by it. When transactions costs are small enough that negotiation is likely if the default rule is inefficient, an injunctive rule is optimal. An alternative to injunction is money damages.

Money damages are harder to negotiate around because the disagreement point to be ordered by a court is hard to gauge *ex ante*; however, money damages are closer to efficient if negotiation fails. When transactions costs are high, liability in money damages makes the anticipated result, litigation, relatively efficient (Calabresi & Melamed (1972)).

Now I apply the Coasian argument to lawyers' common agency rights. In the absence of any explicit duty or contract, the power to enter into common agency naturally lies with the agent since the agent may represent many principals without the principals' knowledge. Accordingly, the focus is on how a principal might negotiate for exclusivity.

The most elegant way to resolve common agency questions would be reliance upon an efficient default rule. It can be efficient for terms to be omitted with the expectation that default rules will apply (Shavell (1984)). In part, perhaps, ethical rules are gap fillers for incomplete contracts, but they are needed to do more than fill gaps. As in *Worldspan*[8], the lawyer might use the retainer agreement to fill gaps from the lawyers' perspective, setting aside an efficient rule. It is not surprising then that ethics rules, unlike gap-fillers, are more often compulsory rules than default rules.

If a default rule will not suffice, four contractual approaches suggest themselves to a principal who seeks explicit control over conflicts: (1) blanket exclusivity, (2) a nuanced conflicts clause, (3) monitoring, and (4) outcome-contingent compensation, such as a contingent fee agreement. Each is addressed in turn:

Straightforwardly, the client may offer money in exchange for blanket exclusivity, but the straightforward approach does not work. *Worldspan* illustrates a common corporate law firm practice: Always assert the right to common agency. The bald assertion that exclusivity is never available is softened by language limiting common agency to cases in which the *lawyer* perceives the other agency to be unrelated and the *lawyer* promises not to

[8]Descriptions of cases cited are given in Appendix C.

use the client's confidential information against the client. One may argue that a sophisticated client may reject this boilerplate and seek more favorable terms. But that is what a sophisticated client tried and failed to obtain in *Worldspan*. The agent's inflexibility is understandable. A specialist law firm becomes a specialist by taking many cases within a narrow field and specialization is inconsistent with exclusivity. Specialization is needed to gain and keep the talent the client seeks, so the specialization interest suggests that the agent should have the right to common agency. For more on specialization and lawyers' attitudes toward conflicts of interest, *see* Epstein (1992).

Alternatively, the client may seek exclusivity for a nuanced subset of the possible conflicts: those conflicts most important to the client and least important to the lawyer. However, recall that the lawyer/client relationship is already derivative; it is about the transactions costs associated with another substantive relationship. Clients may have little information and little *ex ante* interest in remote contingencies in what is already a second order matter. For example, in *Worldspan* the other client was retained several years after the first representation began. *Fiandaca* illustrates another remote contingency realized. Further, lawyers know more than clients about the relationships between lawyers and clients, even sophisticated corporate clients. Lawyers receive training in law school and testing in the bar examination on conflicts of interest. They are much more likely to be cognizant of the issues and naturally offer a conflicts clause that views the problem from the lawyer's perspective. In practice, subject to third-party oversight, lawyers dictate the technical details of the relationship. After all, negotiation and drafting of just this sort of technical detail may be precisely the skill the client seeks. Even if the principal overcomes informational handicaps and obtains a nuanced conflicts clause, its details may depend upon sensitive confidences of the client. A nuanced clause may be unenforceable because verification is too costly to the principal, *see, Levine v. Levine*, 436 N.E.2d 476 (NY 1982) (*Levine*).

Third, the principal can seek monitoring rights and control over conflicts. But monitoring is as unlikely a fix as blanket exclusivity. The principal may not know that the agent represents others and, courtesy of attorney/client privilege, he has no way to find out. And the principal lacks the skill to second-guess the agent's conflicts. Monitoring is not the answer (Macey & Miller (1997)).

Finally, the principal might propose a contract that gives the lawyer the incentive to faithfully represent the client. For example, a client could sell her claims to a lawyer, and let the lawyer pursue it. In common agency problems, the thought experiment is to have all the strategically interacting principals sell their claims to their agent. The lawyer owns the claims and so has every reason to work out the conflicts efficiently. A half measure is a contingent fee agreement, a form of inefficient sharecropping contract.

Aligning incentives fails for both theoretical and practical reasons. Theory first: (1) Contingent contracts shift risk to the agent. The agent is in the business of processing claims, not financing and insuring them, and may be expert at processing claims but ill-equipped to finance and insure them. If we wish to allow specialization, "getting incentives right" is no panacea here. (2) An old saw has it that one who represents himself has a fool for a client, and an idiot for a lawyer. Lawyers are thought to have a role as the independent, cooler, perhaps rational, head. Contingent fees undermine independence and may also interfere with strategic delegation (Vickers (1985)). There may be sound reasons for intentionally separating management and ownership of a claim. (3) If the fee depends on an easily measurable dimension of performance, more subtle aspects of the job are slighted. Under a variety of situations with differences in observability of performance, a fixed fee is optimal (Holmstrom & Milgrom (1992)).

Incentives in the form of contingent fee agreements are also problematical in practice: (1) It is difficult to align an agent's incentives precisely with those of the principal. Executive stock options, for instance, having little risk of loss and leveraged gains, may have encouraged moral hazard during the "millennium" boom. Contingent fee agreements have similar flaws. (2) In many legal matters contingent fees are illegal and claims are not alienable. A criminal defendant cannot sell off his risk of jail time. Many family law claims are inalienable. A license, permit or regulatory proceeding may be specific to the applicant. Equitable claims and remedies are specific to the parties. (3) With exceptions, lawyers' retainer agreements are based on hourly fees, where the hourly rate depends on the lawyer's experience, reputation and marketing skills. Contingent fees do occur on the plaintiff's side of tort cases, class actions and (in effect) in some bankruptcy matters. These plaintiffs are often principals whose ability to pay for legal services is represented by their unliquidated asset. Contingent fees are a financing tool, where the lender is the lawyer. This lawyer/lender is in the best possible position to evaluate the value of the contingent asset, so, in the absence of liquidity, a contingent fee is the optimal way to finance a case. Except in types of cases where contingent fees often act as financing tools of last resort, contingent fees are unusual in practice.

In short, it is unclear how a principal can obtain efficient exclusivity by explicit negotiation or reliance on default rules. On the other hand, some insight is still available from Coase. The role of agents is to accept the delegation of tasks from principals. Necessarily, if delegation is to occur, the value of the principal's task must be larger than the agent's expected value added. When in doubt, since the principal's stakes are likely larger, society should protect the principal. If the agent represents the principal they are in communication and transactions costs may be low. This suggests that the property right be in the principal, as an injunctive right, but subject to waiver. Given the agent's advantages, the agent must have a duty to insure any principal's waiver is informed. That, very oversimplified, is the positive law on conflicts, and about as far as one can get with the Coase Theorem (Macey & Miller (1997)).

The Coasian argument turns on the idea that agents gain from com-

Figure III.5: 2 x 2 Game

R\C	l	r
u	R_{ul}, C_{ul}	R_{ur}, C_{ur}
d	R_{dl}, C_{dl}	R_{dr}, C_{dr}

mon agency and principals lose. It ignores a large set of situations in which the agent is facilitating coordination between principals. Setting that aside, the Coasian argument also does not help distinguish cases in which common agency is efficient from those in which it is overreaching. To get beyond generalities, one must consider the kinds of strategic interactions principals have. A model that takes strategic interaction into account follows.

III.F.2 Game Theory Model.

In this section, I develop a game theory analysis of the types of conflicts principals have.

The 2 × 2 Game

Suppose litigation is a binary choice game between two principals, $i = 1, 2$, Row and Column. The principals choose their strategies simultaneously and independently from a set of profiles, $s \in S$, $S = S_R \times S_C = \{u, d\} \times \{l, r\}$. The principals' contingent payoffs are eight numbers, R_{ul}, C_{ul}, R_{ur}, C_{ur}, R_{dl}, C_{dl}, R_{dr}, and C_{dr}. Utility is linear in payoffs; agents are risk neutral. The strategies would most often represent choices in litigation, and payoffs the outcomes of litigation. The case studies in the main text and Appendix C suggest concrete examples. The game is given in Figure III.5.

These assumptions about the game the principals play are obviously restrictive. Among other things: (1) There may be many principals, and their interaction may be more complex than the simple model here admits[9]. (2) The strategy space may be larger than a binary choice. (3) Preferences may not be consistent with von Neumann-Morgenstern expected utility. (4) Choice may not be simultaneous. (5) Principals may have incomplete or asymmetric information about their game. (6) Principals may not use standard game theory to choose strategies. Nevertheless, there is insight to be gained from this simple model.

Consider in turn three different techniques principals may use to obtain outcomes for their games. These techniques need not lead to equilibria in the usual sense and need not be unique. I call their outcomes solutions. The three techniques are: (1) solutions without common agency (sometimes called "separate agency" or an unmediated solution), for which I assume the solution set is the set of Nash non-cooperative equilibria, (2) mediated equilibrium solutions, and (3) arbitrated solutions. I suppose an agent has full information about the game and may represent both principals. The agent is paid a fixed sum, trivial to the principal, for each agency she undertakes.

First consider separate agency: Let n be a pure Nash equilibrium, N be the set of such equilibria, x be a mixed strategy equilibrium and X its set; finally cN is the convex hull of all Nash equilibria. An upper bound on the set of equilibria which might arise under strictly rational behavior in a stage game is $N \cup X = cN$. However, mixed strategies are problematical in this context. It is appropriate to consider them where interpretation as frequencies in a repeated game could be plausible, perhaps as in *Aetna.* Where repetition is

[9]In the cases in which lawyers' common agency is an issue, there are often many principals. In some the principals are symmetric, like the case of similarly situated defendants. In these cases, treating the number of principals as two may be a harmless simplification. Where the principals are not all symmetric, this model may apply to the relationship between subsets of principals. For example, in a case with a plaintiff and many defendants, the subset of principals to whom the model helpfully applies might be the set of all the defendants. The remaining conflicts, between plaintiff and defendants as a whole, may be analyzed by another, separate, application of the model; in the one-plaintiff many-defendant hypothetical, this second strategic interaction would typically be a simple Matching Pennies Game θ_M. Indeed, this kind of approach is used for all three of the case studies. By contrast, if the strategic interactions are not separable, this model is too simple.

unlikely, as in *Fiandaca*, interpretation as randomization or mixtures in beliefs remains, but makes little sense.

Second, consider mediated equilibria: The agent may offer a strategy profile as public message to the principals. Each principal assumes the other will play according to the profile, and considers the benefits of defecting. In other words, the agent acts as a mediator between the principals. Let m be a mediated solution, which is an equilibrium under principals' perfect common knowledge, and let M be the set of mediated solutions. It follows immediately that $cN = M$; however, again, mixtures may or not be plausible.

Two refinements of solution sets are closely related to the principals' individual rationality constraints: (1) if the principals both prefer one strategy profile to another, so will the agent; and, (2) an agent will only accept an agency on behalf of the principals if she can propose a strategy profile that (strictly) adds value relative to a Nash equilibrium. The first refinement is obvious. This second refinement implements the idea that an agent will act in common agency on behalf of the principals only if she might have something to add through commonality. If she adds precisely nothing, then justification for common agency may come from the agent's specialization interest, but not from the principals' strategic interaction (or perhaps its unverifiable dynamic extension, discussed later). From this distinction about why common agency is to be favored arises a key point: In neutral common agencies, the principals' consent is superfluous. Since principals might withhold consent for inefficient strategic reasons, consent should not be required.

More formally, define ">" over strategy profiles as follows: For $m, m' \in M$, $m > m'$ if $m = (S_R, S_C), m' = (S'_R, S'_C), S_R > S'_R, S_C > S'_C$. Then M^* is defined as follows: If $m \in M$ and there exists an $n \in N$ such that $m > n$ and there is no $m' \in M$ such that $m' > m$ then $m \in M^*$. If M^* is not empty then principals may gain from common agency mediation. Elements of M^* are Pareto optimal among non-cooperative equilibria and strictly preferred to some non-cooperative equilibrium.

Third, consider arbitrated solutions: The agent may impose a strategy profile acting as an arbitrator. Since the agent may be the one who actually plays, by conduct in court or otherwise, an imposed strategy profile need not necessarily be supplemented by explicit third-party enforcement. Let a be such an arbitrator's solution, and A its set, the set of all feasible strategy profiles. Note: elements of A need not be equilibria. By analogy to M^*, A^* is defined as follows: If $a \in A$ and (1) there exists an $n \in N$ such that $a > n$, (2) there is no $a' \in A$ such that $a' > a$, then $a \in A^*$. If A^* is not empty then the principals may gain from common agency arbitration. Note that $M^* \subseteq A^*$[10].

Elements of A^* that require coercion are problematical under ethics rules distinct from the ethics rules at issue here and may also have dynamic consequences; however, a contract interpretation is less troublesome. Since elements of A^* are superior to non-cooperative solutions, principals would be willing to commit *ex ante* to strategy profiles in A^*. The agent's role is then just that of a third-party enforcer, enforcing the principals' fully informed and rational commitment. This interpretation reconciles most of the cases. *Compare Kerry Coal Co. v. United Mine Workers,* 470 F.Supp. 1032 (WDPA 1979) *(Kerry) with Hayes. Hayes* is a well-established but inefficient precedent against delegation of settlement authority even with informed consent.

Even in this simple context, arbitrary payoffs yield an infinite variety of possible games. Games are classified according to whether the game is synergistic, destructive or neutral. Any game with any potential for synergy is treated as synergistic. Games are also classified according to the number of pure strategy Nash equilibria (zero, one or two).

[10]Another candidate for a solution technique is correlated equilibria, in which the agent may provide different messages to the principals and they play a Nash non-cooperative equilibrium conditional on the messages, *see, e.g.,* Fudenberg & Tirole (1991, p. 53). The information an agent provides in connection with correlated equilibria may be incomplete and may differ between principals, but must be accurate so far as it goes. I do not develop this idea here because interesting correlated equilibria require misrepresentation by omission, which raises other ethical issues.

Game Types and Solutions

Each variation in payoffs yields a new game which is characterized by one of five types: $\{\theta_P, \theta_S, \theta_2, \theta_1, \theta_M\}$. These five types are classified in Table III.1 and further defined as follows:

Prisoner's Dilemma θ_P. The Prisoner's Dilemma has one Nash equilibrium, in dominant strategies. $N = \{(d, l)\}$ in Figure III.1. The payoff space graph is another view of strategic interaction. In this graph, the vertical axis is Row's payoff, higher is better for Row; and the horizontal axis is Column's payoff; further to the right is better for Column. Pure strategies are labelled; mixtures of pure strategies are grey. In the Prisoner's Dilemma, the grey area is the set of feasible mixed strategy profiles; in later graphs two shades are used and the darker one is for the convex hull of Nash equilibria labelled cN. In the Prisoner's Dilemma, Figure III.1, profile (u, r) is Pareto preferred to (d, l), but fails as a mediated solution because both principals defect. $M = \{(d, l)\}$; while M^* is the empty set. The arbitrator solution set A^* is the kinked line in Figure III.1. Strategy profile (u, r) is the arbitrator's unique pure strategy solution; but the principals both prefer some mixtures of this strategy with (u, l) or (d, r) to the Nash equilibrium (d, l) and such mixtures are Pareto optimal. Arbitration facilitates extraction of the possible synergies in the Prisoners' Dilemma.

Battle of the Sexes θ_S. The Battle of the Sexes θ_S, has two pure equilibria in Figure III.6. The equilibria are (d, l) and (u, r); Row favors (d, l); Column favors (u, r). The Battle of the Sexes is cooperative in choosing to coordinate on an equilibrium, but competitive in choice between equilibria. If there is no way to coordinate on one of the pure strategy equilibria, the principals may mix strategies—sometimes obtaining one equilibrium or the other, but sometimes failing to coordinate, obtaining an inferior mixed strategy equilibrium. In this game there is a unique x in the set of mixed strategy equilibria

X. If one combines these three equilibria in deterministic proportions, then the triangular space in Figure III.6 is the convex hull of Nash non-cooperative equilibrium outcomes cN.

Mediators and arbitrators can deliver any pure equilibrium and mixed strategy equilibria on the line between (4, 3) and (3, 4). $M^* = A^* = \{(d, l)p + (u, r)(1 - p) | p \in [0, 1]\}$. Solutions off this line are either not feasible or Pareto dominated. Principals are competitive over solutions on the line.

Mixed strategies have different implications in Battles of the Sexes θ_S than in Matching Pennies θ_M. In Matching Pennies θ_M the principals seek unpredictability by mixed strategies. There is no cooperative solution because predictable behavior can be exploited without risk of effective retribution. By contrast, in Battles of the Sexes θ_S, mixed strategies are more ambiguous. Randomizing may be feasible, but yields the Pareto inferior solution x in Nash equilibrium. Mutually superior results can be obtained by deterministic and cooperative play such as alternating between equilibria. This provides additional insight on why mediation and arbitration helps in mixed strategy Battles of the Sexes, but not mixed strategy Matching Pennies: Principals can randomize, and so do as well as possible in Matching Pennies on their own; only in a multiple equilibrium game like Battle of the Sexes does coordination have a role. Sonsino (1997) treats coordination in mixed strategies.

Stag Hunt θ_2. A Stag Hunt θ_2 (Figure III.2) is also a game with two equilibria, but they are Pareto ranked. The pure equilibria are (d, l) and (u, r), but (d, l) is better for both principals than (u, r). There is an element of cooperation, but no element of competition. As with the Battle of the Sexes θ_S, there is also a unique mixed strategy equilibrium X. The convex hull of equilibria, a line in Figure III.2, is the Nash equilibria set cN. Here, $M^* = A^* = \{(d, l)\}$.

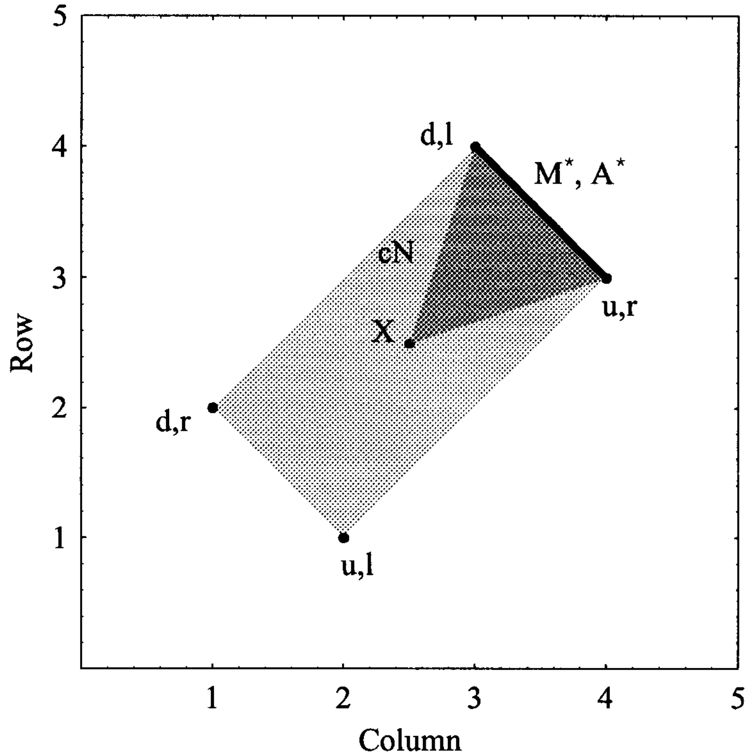
Although not a distinct type in my catalog, it is possible to have two equilibrium games, Battles of the Sexes θ_S or Stag Hunts θ_2, combined with Prisoner's Dilemmas θ_P. Aumann used one such game (Figure III.7) to

Figure III.6: Battle of the Sexes θ_S

R\C	l	r
u	2	4
	1	3
d	3	1
	4	2

Payoff Space

N—pure Nash equilibrium (2 points)
X—mixed Nash equilibrium(point)
cN—convex hull of Nash equilibria (triangle)
M^*—mediator equilibrium (line)
A^*—arbitrator solution (line)



illustrate correlated equilibrium, Fudenberg & Triole (1991, p. 54). This one is a Battle of the Sexes because Row prefers the (u, l) equilibrium and Column prefers the (d, r) equilibrium. Figure III.7 also has the flavor of a Prisoner's Dilemma because (d, l) is not an equilibrium, but is attractive as a cooperative solution. An arbitrator could mix any of $(d, l), (d, r)$ and (u, l). A mediator can mix (d, r) and (u, l). An arbitrator adds value relative to a mediator in that for any $m \in M^*$ there is an $a \in A^*$ such that $a \geq m$ with strict inequality for mixed strategies. These mixtures have a cooperative interpretation like Battles of the Sexes θ_S mixtures.

One Equilibrium θ_1. Games having one equilibrium which are not Prisoner's Dilemmas θ_P are all neutral and have a single Nash equilibrium, in pure strategies; M and A are singletons equivalent to N; M^* and A^* are empty. There are several variations on this theme. The case for common agency in neutral games is the agent's specialization interest.

Suppose $C_{ul} = C_{ur}, C_{dl} = C_{dr}, R_{ul} = R_{dl}$, and $R_{ur} = R_{dr}$. The result is the Trivial One Equilibrium Game θ_1. Figure III.8's one Nash equilibrium is $n = (d, r)$. There is no strategy profile that both parties prefer to n, so M^* and A^* are empty. Since Row's choice has no effect on Column, and Column's choice has no effect on Row, there is neither cooperation nor competition between the principals. The argument for common agency is the agent's interest in specialization, not an interest of the principals. While the trivial game *is* trivial, it matters because of the agent's interest, and so requires explicit treatment.

In some other One Equilibrium Games, there is strategic interaction, and equilibrium in dominant strategies, but no conflict. Figure III.9 is a Win/Win Game in which interests are precisely aligned.

Dominant strategy games need not lead to such happy results. In Figure III.10 the dominant strategy equilibrium is (d, l). Column would prefer any strategy profile in which Row plays u, but Row never plays u. This game

Figure III.7: Aumann's Game θ_2

R\C	l		r	
u	5	1	0	0
d	4	4	1	5

Payoff Space

N—pure Nash equilibria (2 points)
X—mixed Nash equilibrium (a point)
cN—convex hull of Nash equilibria (triangle $(d,r),(u,l),X$
M^*—mediator equilibria (line, $(d,r),(u,l)$)
A^*—arbitrator solution (kinked line, $(d,r),(d,l),(u,l)$)

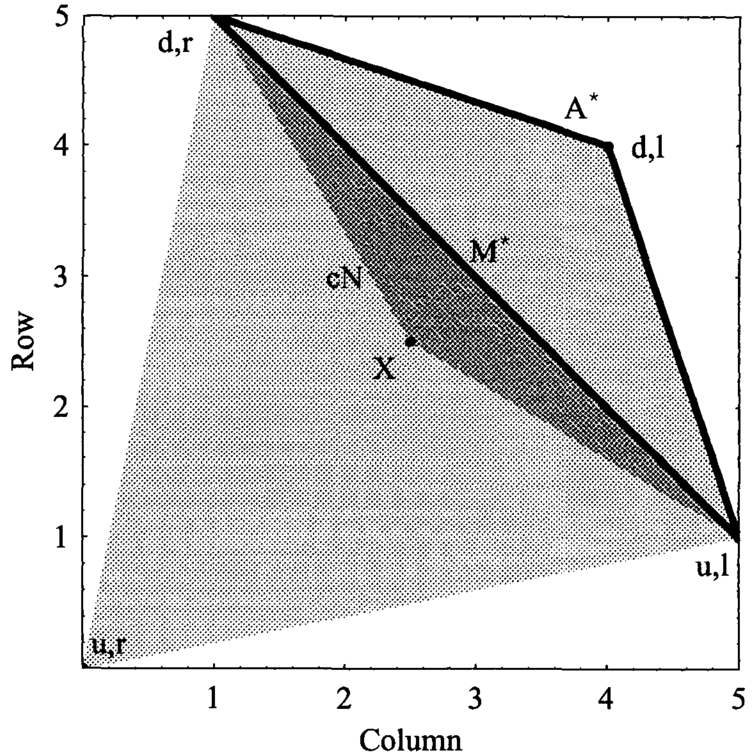


Figure III.8: Trivial One Equilibrium Game θ_1,

R\C	l	r
u	1, 1	1, 2
d	2, 1	2, 2

Payoff Space

N—pure Nash equilibrium (point)
M^*—mediator equilibrium (empty)
A^*—arbitrator solution (empty)

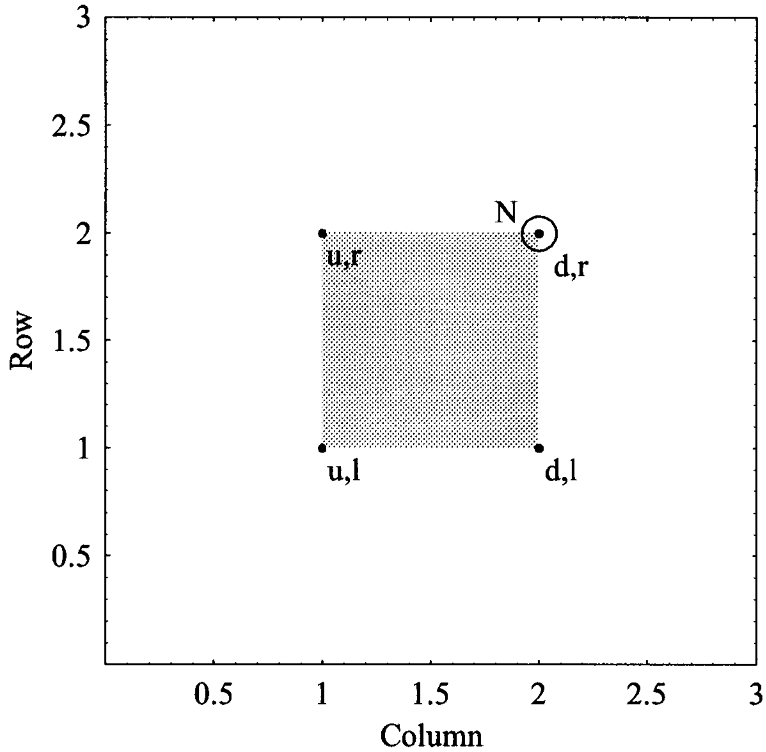
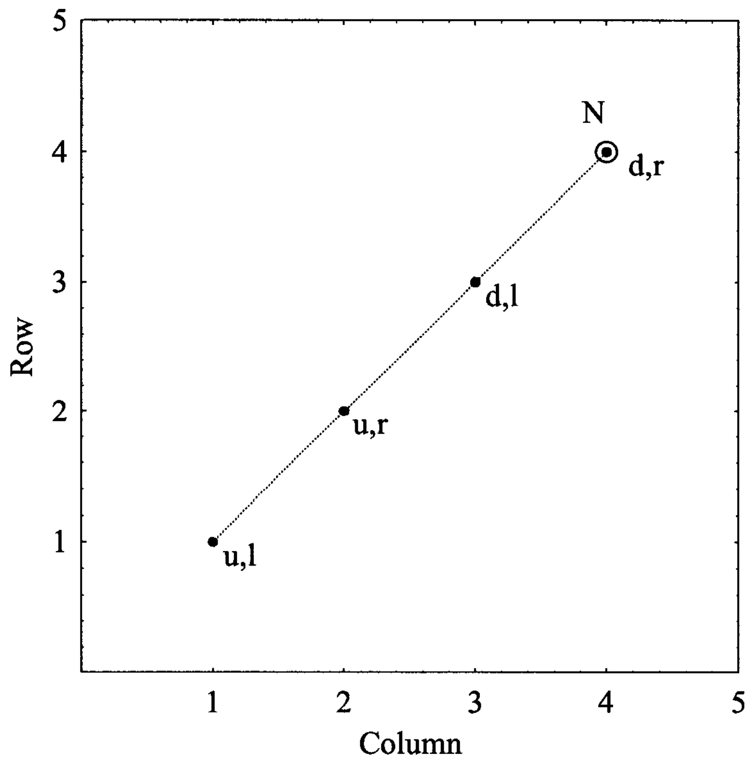


Figure III.9: Win/Win θ_1

R\C	l	r
u	1, 1	2, 2
d	3, 3	4, 4

Payoff Space

N—pure Nash non-cooperative equilibrium (a point)
M^*—mediator equilibrium (empty)
A^*—arbitrator solution (empty)



is a minor generalization from *Universal*, Figure III.4. The generalization is that the payoffs need not be tied.

In another group of One Equilibrium games, the equilibrium is found through iterated deletion of dominated strategies. One player has a favorite strategy, and conditional on that choice, the other player's strategy is determined. Figure III.11 illustrates this case.

(Perturbed) Matching Pennies θ_M. If the game has no pure strategy equilibrium at all, it is a Perturbed Matching Pennies (or just Matching Pennies) game θ_M, Figure III.3. Matching Pennies is the canonical lawsuit where whatever the Plaintiff wins comes from the Defendant in a constant sum contest. For any proposed mediated solution, one principal defects. Therefore, M and M^* must be empty. Any feasible strategy profile could be imposed by an arbitrator; however, none is preferred by *both* principals to any other solution; *a fortiori* no arbitrator's solution is preferred to any Nash equilibrium. A^* is empty. Common agency is destructive in Matching Pennies in that the commonality adds nothing and requires the agent to betray one of her principals by adopting a strategy profile that is certainly to the detriment of one of the principals.

One criticism of this position is: It is the game the creates the conflict, not the common agency. The common agency does not make things worse. This position refuses to take the lawyer's duty of zealous representation and lack of equilibrium in pure strategies seriously. In the pure strategy interpretation of Matching Pennies, there is always an unplayed best response. This appears strikingly in *Fiandaca*, but one might argue that *Fiandaca* is exceptional. However, the problem comes up routinely when the agent represents one client and then adds the conflicting client, as in *Worldspan* and *Dresser.* In these cases, according to the first client and the court, there is no way to add the second client and act consistently with the second client's interests without betraying the first client.

Figure III.10: Dominant Strategy θ_1

R\C	l	r
u	1, 4	2, 3
d	4, 2	3, 1

Payoff Space

N—pure Nash non-cooperative equilibrium (a point)
M^*—mediator equilibrium (empty)
A^*—arbitrator solution (empty)

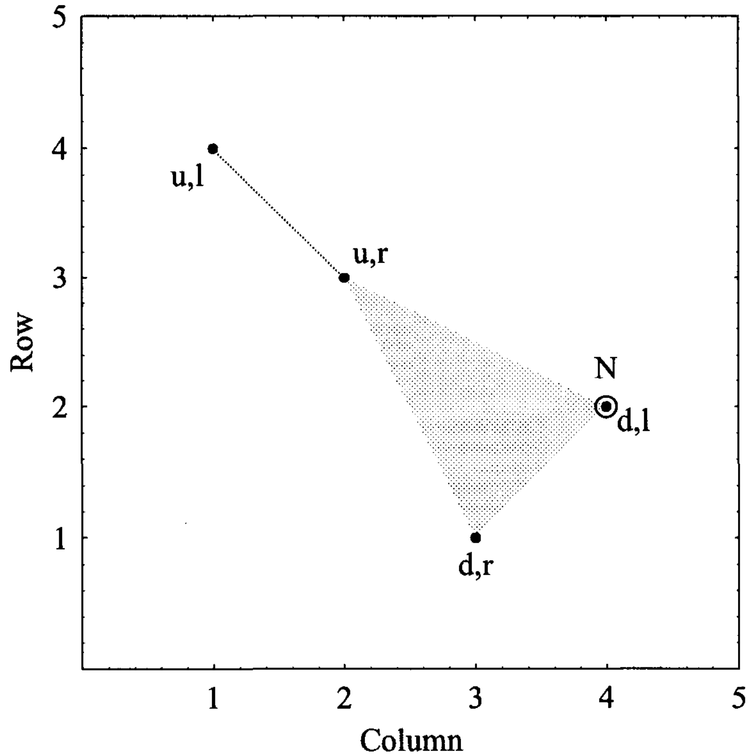
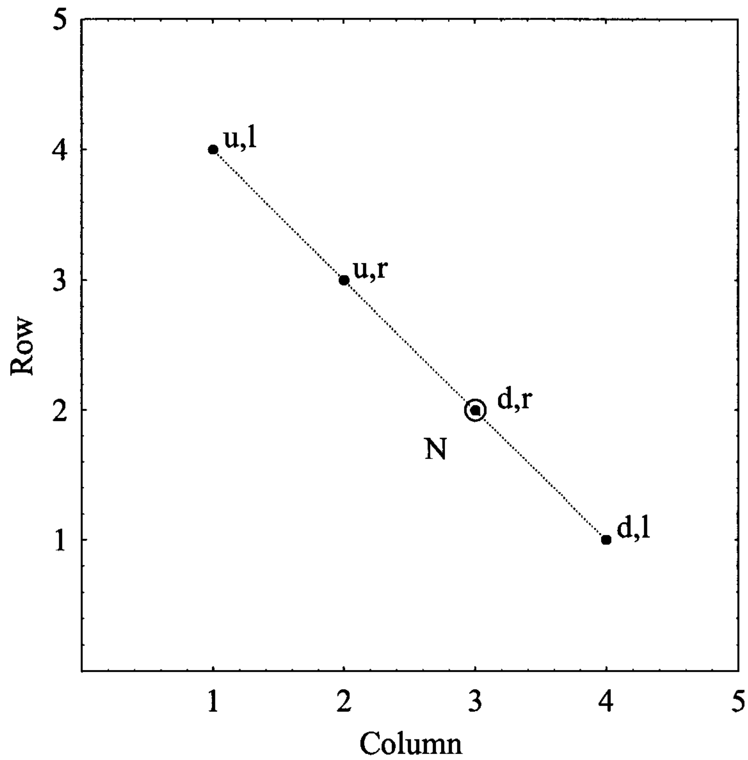


Figure III.11: Iterated Deletion of Dominated Strategies θ_1

R\C	l	r
u	1, 4	2, 3
d	4, 1	3, 2

Payoff Space

N—pure Nash non-cooperative equilibrium (a point)
M^*—mediator equilibrium (empty)
A^*—arbitrator solution (empty)



Another criticism is about mixed strategies: The idea that an agent randomizes rather than reasons out a strategy may be silly, as in *Fiandaca*; nevertheless, an agent could choose randomly and thereby adopt a mixed strategy[11]. Allowing mixed strategies does not change the result. The unique mixed strategy Nash equilibrium in Figure III.3 comes from Row playing u with probability 0.25 and Column playing l with probability 0.5. If mixed strategies are permitted, M^* and A^* can be re-defined as follows: If $m \in M$ and there exists a mixed strategy Nash equilibrium $x \in X$ such that $m > x$ and there is no $m' \in M$ such that $m' > m$ then $m \in M^*$. A^* is re-defined analogously. M^* and A^* thus redefined are still empty[12].

Summarizing Types

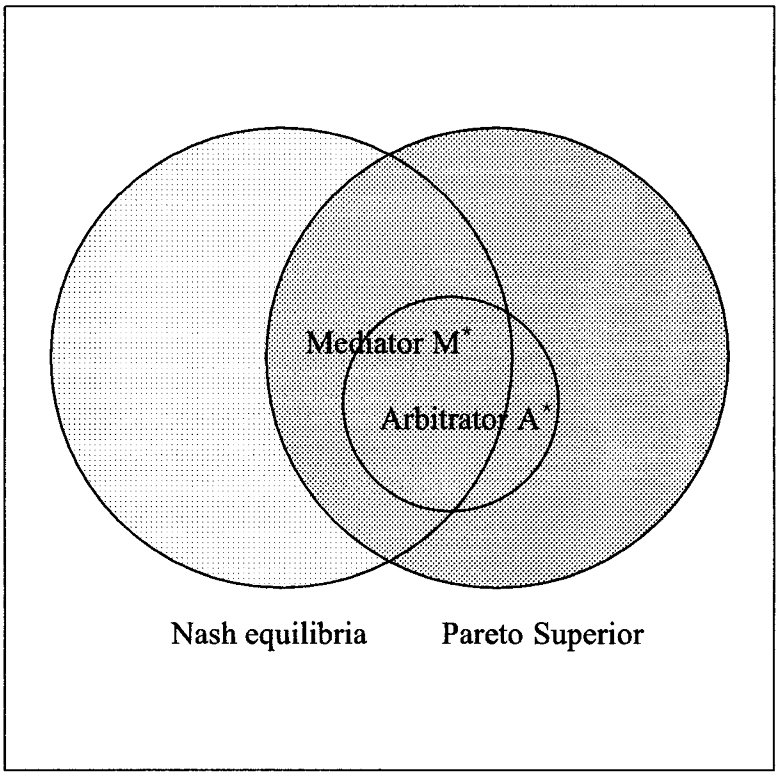
In Matching Pennies θ_M and One Equilibrium Games θ_1, M^* and A^* are empty; and so common agency does not help principals. In the Prisoner's Dilemma θ_P, M^* is empty but common agency arbitration A^* can be helpful. In two equilibrium games, Battles of the Sexes θ_S and Stag Hunts θ_2, both mediators M^* and arbitrators A^* have the potential to add value, but the arbitrator solutions can be superior, as in Aumann's game. Every type which could occur in the simple framework of the model is captured by one of these types.

Figure III.12 shows the relationships between solutions. Nash solutions are equilibria. Mediator equilibrium sets M^* are Pareto superior and equilibria. Arbitrator solution sets A^* need not be equilibria at all, but must be Pareto superior.

[11] The choice could be deterministic yet unpredictable, an idea closely related to "purification", Fudenberg & Tirole (1991, p. 236). A deterministic yet unpredictable choice would be unrelated to the merits, hence as objectionable as randomization.

[12] Proof: Suppose the agent chooses $a = (S_R^a, S_C^a)$ and the mixed strategy equilibrium is $x = (S_R^x, S_C^x)$ where S_R is the probability of playing u and S_C is the probability of l. If $a \neq x$ then a is not preferred by both principals to the mixed Nash profile x, which is the only element in X. If $a = x$ then it is not strictly preferred to any $x \in X$, and again fails to be in A^*. The case for M^* is strictly analogous.

Figure III.12: Agent Solutions



III.F.3 Dynamics and Reputation

The normal-form game theory model shows that conflicts of interest can be synergistic or destructive, but there is no pressure that pushes agents toward efficient treatment of conflicts. If, however, a lawyer values her reputation, then the "shadow of the future" may be a simple dynamic which tends to drive agent behavior toward efficient outcomes. A highly stylized model of how this might happen follows.

Suppose an agent represents a pair of principals. The agent's fee will be p. If she declines to represent one of the principals she earns a fee of $p/2$. In types θ_P, θ_S and θ_2, common agency adds value, assuming consent after disclosure. In θ_1 common agency is neutral from the viewpoint of principals *ex ante*. Call all these game types "non-destructive games". Suppose that the probability that a given common agency is a non-destructive game type is $\alpha \in (0, 1)$.

If the agent accepts every common agency, in period t her earnings are p_t. If the agent acts in common agency only for non-destructive games, her expected earnings in period t are

$$\begin{aligned} E[p_t] &= \alpha p_t + (1-\alpha)p_t/2 \\ &= p_t(1+\alpha)/2 \end{aligned}$$

A rational agent will give up common agency only if the gains from separate agency exceed the losses. Since $\alpha < 1$ period t earnings are always reduced by acting ethically so the agent has no contemporaneous incentive to respect conflicts.

Now add dynamics. Suppose each agent represents an infinite series of pairs of principals one after the other in time, $t = 0, 1, \ldots$. No principal has any strategic interaction with any principal other than the principal with which he is paired. Fees vary from pair to pair. The agent earns a fee of p_t for each pair, having i.i.d. distribution $F[p]$, if she acts for both. If she declines to represent one of the principals, she earns of fee of $p_t/2$. Her discount rate is

$\delta \in [0, 1)$. If the agent accepts and is allowed to accept every pair of principals, her lifetime earnings are:

$$\pi = p_0 + p_1\delta + p_2\delta^2 + \ldots.$$

Her expected lifetime earnings are:

$$E_0[\pi] = p_0 + \frac{E[p]\delta}{1-\delta}.$$

If she is always ethical, her lifetime earnings are

$$E_0[\pi^E] = p_0(1+\alpha)/2 + \frac{E[p]\delta(1+\alpha)}{2(1-\delta)}.$$

After the fee is paid, the type of the case and decisions of the agent become common knowledge. An agent who has acted unethically receives no more offers of representation. This unethical agent's earnings are

$$\pi^U = p_0 + p_1\delta\alpha + p_2\delta^2\alpha^2 + \ldots$$
$$E_0[\pi^U] = p_0 + \frac{E[p]\alpha\delta}{1-\alpha\delta}.$$

Ethics pays when $E_0[\pi^E] - E_0[\pi^U] > 0$, or

$$p_0 < \frac{E[p]\delta}{(1-\alpha)}\frac{(1+\alpha\delta)}{(1-\alpha\delta)}.$$

Suppose half the cases are non-destructive games so $\alpha = 0.5$. And suppose the agent handles ten cases per year and the interest rate is 10% per annum (or 1% per case) so $\delta = 0.99$. Then the agent acts ethically unless the value of the current fee is about 6 or more times the average fee. The reputational dynamic deters most violations, since cases in which the fee is at least six times the average fee must be unusual. On the other hand, if the variance in fees is large, the inequality does not always hold, and reputation alone is insufficient to deter strictly rational agents over the course of a career.

III.G Appendix: Law

III.G.1 The Law of Legal Discipline

While called "ethics" and "rules" or even "canons," the law of legal discipline has teeth. It is explicitly stated as compulsory rules, not admonitions or exhortations, and enforced with vigor.

The lead regulator of a lawyer's conduct is the highest court of a state in which the lawyer has been admitted to practice. Commonly, the court delegates most of its supervisory function to its state bar association. Federal courts have a bewildering array of separate admission and regulatory structures. Except for some federal employees in some federal courts, admission to a state bar is a predicate to federal practice, and most discipline is at the state level.

Each state has adopted a set of explicit ethical rules to govern lawyers conduct. The rules that govern lawyer conduct have four sources, statutory law, case law, rules of court and ethical rules, as follows: Each state's general laws contain provisions affecting lawyer conduct. For example, state law of evidence may describe attorney-client privilege. Published opinions of courts make law or interpret statutes. Case law may fill out the contours of a statute by applying it to a specific set of facts. Some opinions are issued by legal regulatory agencies, such as disciplinary review boards. Each court may have rules of procedure which govern conduct in that court.

Finally, and the primary focus here, each state has a formal body of ethics rules. In most jurisdictions these are based on codifications adopted by the American Bar Association ("ABA"), either the *ABA Model Code of Professional Responsibility ("Model Code")* or the (newer) *ABA Model Rules of Professional Conduct, 2003 Edition ("Model Rules").* Some states' rules, California prominently, are not closely modelled on either ABA standard, and many others have adopted ABA standards, with variations. Here the starting point is the Model Rules, and despite state variations of detail, for our purposes, the rules are homogeneous.

The most common remedy for an improper conflict is disqualification. The lawyer must drop one or more of the agencies. If a client has been harmed, a malpractice claim may arise. If the conduct is egregious, the lawyer may be disciplined—even disbarred—and forfeit her fees. (*Restatement Third, The Law Governing Lawyers* §128, Comment a.) Disbarment is a severe sanction because it ends a lawyer's career and destroys her human capital. Choice among remedies may by non-trivial. In *Universal* the court refused disqualification because of its potential for collateral damage, but suggested a disciplinary proceeding.

III.G.2 Law on Conflicts of Interest

Lawyers' conflicts is a large field. The focus here is the representation of two or more concurrent clients in civil litigation. Even here, however, there are conflicts with many different characters. The obvious case is representation of a plaintiff and a defendant at the same time. A few of the kinds of cases that raise more subtle issues are: multiple plaintiffs (or multiple defendants), insured and insurer, employer and employee, principal and guarantor. Further, the principals may have an "economic" conflict. Representation of a labor union against management in one dispute, particularly if it may set a precedent, arguably affects the ability of an agent to represent a different management against a different labor union. Similar economic conflicts may arise between competing firms and between principals separated by other policy disagreements, like environmentalists and industrialists.

Model Rule 1.7 provides:

(a) Except as provided in paragraph (b), a lawyer shall not represent a client if the representation involves a concurrent conflict of interest. A concurrent conflict of interest exists if:

   (1) the representation of one client will be directly adverse to another client; or

(2) there is a significant risk that the representation of one or more clients will be materially limited by the lawyer's responsibilities to another client, a former client or a third person or by a personal interest of the lawyer.

(b) Notwithstanding the existence of a concurrent conflict of interest under paragraph (a), a lawyer may represent a client if

(1) the lawyer reasonably believes that the lawyer will be able to provide competent and diligent representation to each affected client;

(2) the representation is not prohibited by law;

(3) the representation does not involve the assertion of a claim by one client against another client represented represented by the lawyer in the same litigation or other proceeding before a tribunal; and

(4) each affected client give informed consent, confirmed in writing.

Other rules address other aspects of concurrent conflicts. Taken together, the law creates a triad of characterizations.

(1) *No conflict.* A lawyer may have more than one client if there is no conflict of interest between them. If there is no "concurrent conflict" between them, analysis under this rule is done.

(2) *Waivable conflict.* There may be a concurrent conflict, but it can be waived if each affected client gives informed written consent.

(3) *Direct conflict.* There are direct concurrent conflicts that cannot be waived—a true limitation on the freedom of contract between attorney and client. Direct conflicts include conflicts without bright lines—where the lawyer's ability to represent both clients would be compromised in some fashion. Other law prohibits conflicts where one client is a government, or limits the authority of governments to waive conflicts. Also, some states absolutely prohibit some common agencies, such as representation of more than one defendant in capital cases (*Model Rule 1.7, Comment 16.*)

Representation of opposing parties in litigation is flatly prohibited (*Model Rule 1.7(b)(3)*). On the other hand, if the *client* is the conflicted one, the agency may be proper, *MGIC*.

A lawyer has conflicts of interest when her clients are "adverse." According to the *Restatement Third, The Law Governing Lawyers* §*128, Comment b.*, four values are packed into the term "adverse", which I have labelled "confidentiality," "loyalty," "process integrity," and "coordination."

(1) *Confidentiality.* First is the client's interest in confidentiality. One client's secrets should not be used to further the interests of another client. Such abuses are hard to detect and therefore require preventative measures. In representing a client a lawyer may learn more than the strengths and weaknesses of a particular case. A lawyer may learn about negotiating strategies, peccadillos, internal processes and other quirks of the client that could be used against them; however, the client also knows those things. If the only value implicated in the conflict is confidentiality, it should be a waivable conflict. The client has a good basis to assess whether the lawyer has, or will obtain, any important secrets.

(2) *Loyalty.* The client should have confidence in the lawyer's zealous fidelity to the client's interests. An example arises when a lawyer represents A against B and B against C in an unrelated case.

Parties can be adverse, even though the lawyer's representation of them is not.

> "Thus, absent consent, a lawyer may not act as an advocate in one matter against a person the lawyer represents in some other matter, even when the matters are wholly unrelated. The client as to whom the representation is directly adverse is likely to feel betrayed, * * * ." (*Model Rule 1.7, Comment 5*)

The law distinguishes between adverse parties and adverse ideas. The law does not recognize "economic" conflicts as adverse (*Model Rule 1.7,* *Comment 6.*) There is no ethical constraint on representing Coke and Pepsi at the same time in distinct cases, nor in representing environmentalists against one oil company in one case while representing another oil company against other environmentalists in another. But the distinction between a client and his positions gets muddy, *see, Model Rule 1.7, Comment 24; Restatement §128, Comment f, Illustrations 5 and 6.*)

As with confidentiality, adverse interests associated with loyalty should be waviable conflicts. The client can assess whether another specified representation is questionable to the client. Here, however, another problem arises: Large clients may also act tactically in their relations with law firms, seeking to retain all the most reputable ones so that their potential enemies are at a disadvantage. If a client's refusal to waive the conflict is tactical, the court may override it, *Universal.*

(3) *Process Integrity.* Tribunals want assurance that their processes, which depend on a contest of vigorous advocacy, are not compromised. Since the holder of this interest is not the client, the client cannot waive such conflicts.

(4) *Coordination.* Fourth, clients may want to economize on transactions costs, and gain benefits of coordination by combining their positions.

Parties may present a mixture of adverse and common interests. A group starting a business is a classic case. They are adverse in deciding how the burdens and benefits are to be divided, but have common interests in joining their resources to maximize the value of the project. Close and difficult questions arise here. (*Model Rule 1.7, Comment 8.*) "The question is often one of proximity and degree" (*Model Rule 1.7 Comment 26.*) "[C]ommon representation will almost certainty be inadequate if one client asks the lawyer not to disclose to the other client information relevant to the common representation," (*Model Rule 1.7, Comment 31*), implicating the confidentiality value.

III.H Appendix: Cases

In the following I have simplified the facts and sometimes changed the vocabulary to fit the game theory paradigm. The cases are in alphabetical order. Each case is summarized, and followed by an analysis of its type, and determination whether the outcome is efficient in terms of the game theory model in section 3 and consistent with precedent as embodied in the *Restatement.*

***Aetna Cas. & Sur. Co. v. United States*, 570 F.2d 1197 (4th Cir 1978) (*Aetna*).**

See main text.

***In re Dresser Ind.*, 972 F.2d 540 (1992) (*Dresser*).**

Susman Godfrey was lead counsel for Dresser in two major cases, one concerning asbestos contamination and a second making antitrust claims about the compressor market in Saudi Arabia. Then came a third case: Stephen Susman wrote Dresser saying he was suing Dresser; specifically, Susman would be chairman of the Plaintiff's committee in a price fixing action against all major drill bit makers, of which Dresser was one. Susman offered to resolve the conflict by withdrawing from the cases in which it represented Dresser. Instead, Dresser objected to Susman's representation of the Plaintiff's committee.

Although the court clothed its opinion otherwise, it repeatedly and emotionally returned to the loyalty interest: "A lawyer should not sue his client." The court backed its position with the concern that a lawyer might pursue one case less vigorously out of deference to the client, but that obviously was not what was going on in this case.

On the other hand, Susman lawyers "have had relatively unfettered access to data concerning Dresser's management, organization, finances, and accounting practices. Susman Godfrey's lawyers have engaged in privileged communications with Dresser's in-house counsel and officers in choosing antitrust defenses and other litigation strategies." If there is a sound argument for disqualification, this is it. In the course of its work for Dresser, Susman learned general information about the workings of Dresser that might be of use to it in suing Dresser. Dresser's strongest objection to Susman could be based on what Dresser knows Susman learned in privileged communication. The case should turn on the confidentiality interest. The result was right; Susman was disqualified, but the grounds were wrong.

Type, Matching Pennies θ_M. Dresser's lawyers are suing it. If Dresser's lawyers do a good job in suing it, it suffers. From Dresser's perspective this violates an individual rationality constraint. The issue here, as in the other matching pennies types, is whether the two representations are closely enough related to find strategic interaction. By finding that the confidentiality interest is at stake, the court is finding the representations too closely related.

Efficient: Yes. The court denied common agency as a violation of individual rationality constraint in Matching Pennies.

Consistent: Yes. The outcome is consistent with reliance on the confidentiality interest, though the analysis is flawed.

***Dunton v. County of Suffolk*, 729 F.2d 903 (2nd Cir 1984) (*Dunton*).**

Emerson escorted Angela to her car after an office party and "improper advances" ensued. Robert, Angela's husband and a police officer, came upon them in his patrol car and assaulted Emerson. Angela filed a criminal complaint against Emerson for sexual assault; Emerson sued for 50 million dollars for his beating and a subsequent alleged cover up. Robert received a boilerplate form about possible conflicts of interest when represented by County Counsel, and signed it. At trial Counsel's theory of the case was that Robert acted as a husband, not a County employee and the County was not responsible. The jury awarded a judgment against Robert the husband for

$20,000, but not against Robert the employee or against the County. Robert now recognized the conflict of interest and sought redress. On appeal, he won; the court returned the case for a new trial.

The defense had two plausible strategies from which to choose: (1) Robert as policeman was acting in good faith under color of law, and (2) Robert as husband was justified in defending his wife.

Type: Battle of the Sexes, θ_S. The defenses are inconsistent and represent two equilibrium strategies for the defense. The County prefers (2). Robert's best chance may be (1). If Robert and the County have separate counsel, both defenses may be offered, and raise the likelihood that both are rejected. It is a Battle of the Sexes.

Efficient: Yes. Common agency could have been efficient here. If Robert understood the game he could have participated in the choice of equilibrium; however, in the absence of informed consent, common agency violated his individual rationality constraint and common agency is inefficient.

Consistent: Yes. Waiver of the conflict was not fully informed.

***Fiandaca v Cunningham*, 827 F.2d 825 (1st Cir 1987) (*Fiandaca*)**

See main text.

***Hayes v. Eagle-Picher Ind. Inc.*, 513 F.2d 892 (10th Cir 1975) (*Hayes*)**

O'Keefe represented a group of 18 plaintiffs. There was a dispute over whether they had agreed to resolve disputes among themselves by majority rule. When a settlement offer was accepted 13 to 5, the dissenters objected. The court ruled that the plaintiffs could not bind themselves to coordination by majority vote. The settlement was reversed. Authority to settle is personal and inalienable.

Type: Battle of the Sexes θ_2. The *Hayes* principals are aligned in wanting to make the pie as large as possible. They may differ on how to divide it. The court is not explicit, but it seems that the issue was not how to divide the settlement, but whether to take it. The principals may also differ on whether the settlement proposal maximizes expected utility for two reasons: (1) they assess the expected outcome of litigation differently, (2) they have different attitudes toward risk. There are two equilibria, trial or settlement, and the parties learned *ex post* that they were unranked. The plaintiffs agreed *ex ante* to choose among them by majority vote. This is a Battle of the Sexes θ_S.

Efficient: No. The court allowed parties to contract to delegate settlement authority, but not to make an irrevocable commitment. The result is inefficient, and bad law.

Consistent: No. The result is well-established as a common law rule, but acknowledged to be inconsistent with the legal interests analysis, *Restatement* §22(3).

***In re Houston*, 985 P.2d 752 (NM 1999) (*Houston*).**

Wife initially presented the marital separation as uncontested. Houston represented Wife in the dissolution, and advised her to file a domestic violence petition, apparently commonly done without legal representation. Wife wanted, but Houston did not seek, restrictions on Husband's visitation with the children. Houston represented Husband in the criminal prosecution that was triggered by the domestic violence petition. Husband was prosecuted for molestation of his daughter and battering of his wife; Husband pled guilty. There is more, but that will do. This was not a close case and Houston was severely sanctioned, but it does emphasize the incentive divorcing principals feel to feign a more cooperative posture than exists to avoid escalating hostilities and to minimize costs.

Type: θ_M, Matching Pennies. In *Houston*, Wife described the legal dispute as a two equilibrium game, contested divorce and uncontested divorce, similar to *Klemm.* In fact, Husband was physically abusive and contested divorce was inevitable, which in turn is a Matching Pennies game, θ_M.

Efficient: Yes. The lawyer was severely disciplined for common agency.

Consistent: Yes.

***Hurt v. Superior Court,* 601 P.2d. 1329 (Ariz. 1979) (*Hurt*).**

Dickinson died in a house fire on the morning of his wedding. Six months later his alleged son was born. The son and Dickinson's mother sued Dickinson's landlord for negligence in maintaining a furnace. The landlord claimed the son and grandmother had a conflict of interest, in part because the son's status was questioned. The court found no conflict, though it was concerned that the infant child might need special consideration.

There are a couple of ways to think about a wrongful death claim. One is harm to the decedent, and appointment of some sort of agent for the decedent's claim. Another is to consider the loss incurred by those who knew the decedent. The law of the case was that the claim was for harm to the decedent, but compensation was not paid to the decedent (or the decedent's estate). Instead, compensation is divided among certain relatives according to their loss.

Mother and son have joint interests in extracting compensation from the defendant. The poignant story about Dickinson's son might increase the value of the case to Mother, and Mother's presence might help the infant with the business of bringing a case; common agency could reduce the cost of presenting both claims. On the other hand, they have a conflict about how to split the proceeds or how to suggest the jury make an award, if that is an issue for the jury. It appears that the court substituted its judgment on behalf of the son. Son was better off with Mother's lawyer than on his own, in the court's opinion.

Type: θ_S, Battle of the Sexes. In *Hurt* the principals share a wrongful death claim against a third party. Like *Hayes* they share in wanting to make the claim as large as possible, while having a conflict in how to share it. The twist in this case is that one of the principals is an infant.

Efficient: Yes. The court allowed common agency.

Consistent: Yes. It is not a "direct" conflict; the difficulty is in ensuring the infant has appropriate bargaining weight in choosing the equilibrium.

***Ishmael v. Millington*, 50 Cal.Rptr. 592 (1966) (*Ishmael*).**

Wife claimed malpractice because the lawyer had not obtained informed consent to common agency in an uncontested divorce. There could have been no informed consent because the lawyer did not tell Wife about all Husband's assets, perhaps because the lawyer did not bother to ask Husband about them. The principals rely upon the professional to tell them what the game is, without that information, they are in no position to judge whether they should consent to common agency in a synergistic game.

Type: θ_2, Battle of the Sexes. The twist is that Wife had inadequate information. She could not assess the proffered equilibrium.

Efficient: Yes. The court would have allowed common agency, if both the principals agreed after full disclosure. The case turns on failure of full disclosure.

Consistent: Yes.

***Kerry Coal Co. v. United Mine Workers,* 470 F.Supp. 1032 (WDPA 1979) (*Kerry*).**

Kerry sued its workers, union and union representatives for an illegal strike. All the defendants were represented by the firm of Kuhn, Engle and Stein ("KES"). Kerry alleged that some defendants would cast blame on others, and so a conflict of interest existed. KES argued that it had fully and fairly discussed theories of the case with its clients and they had agreed to present a united front, not trying to point fingers amongst themselves.

Here the court accepted the claim of waiver after full and fair disclosure and declined to allow the principals' common enemy to force their division.

Type: θ_P, *Prisoner's Dilemma.* This case is a classic Prisoner's Dilemma. It is likely that some of the defendants could have cast blame elsewhere with greater likelihood of success than KES's odds beating the entire suit. But KES acted to enforce an arbitrator solution. The risk of coercion of some defendants by others is large, but if there truly was full disclosure and free agreement to common agency, the result is efficient.

Efficient: Yes. Common agency avoided inefficient defection in a Prisoner's Dilemma.

Consistent: Yes.

***Klemm v. Superior Court,* 142 Cal. Rptr. 509 (1977) (*Klemm*).**

Dale and Gale Klemm asked Bailey, a lawyer and friend, to divorce them. They had two minor children and no assets. Bailey agreed to represent them both, without pay. They agreed to joint custody of the children and no child or spousal support. The common agency was allowed, and the divorce granted. Gale was on Aid to Families with Dependent Children ("AFDC"). The judge accordingly referred the case to the Family Support Division ("FSD"). FSD recommended that Dale be ordered to pay support, and pay it to FSD to reimburse FSD for payments made to Gale. Bailey represented Dale in opposing FSD.

The trial court noted that Gale might not be on AFDC forever, and then any court ordered support would come to her, creating a direct conflict of interest and barring common representation. The appellate court said:

> While on the face of the matter it may appear foolhardy for the wife to waive child support, other values could very well have been more important to her than support—such as maintaining a good relationship between the husband and the children and between the husband and herself despite the marital problems—thus avoiding

> the backbiting, acrimony and ill will which the Family Relations Act of 1970 was, insofar as possible, designed to eliminate. It could well have been if the wife was forced to choose between A.F.D.C. payments to be reimbursed to the county by the husband and no A.F.D.C. payments she would have made the latter choice.

Dale and Gale could waive the conflict in connection with the current AFDC litigation, if fully informed.

Type: θ_2, *Stag Hunt.* In *Klemm* there are two equilibria, litigious divorce and amicable divorce. In this case, it is clear that both principals had a favorite equilibrium, and it was the same one.

Efficient: Yes. Denying common agency here would have forced a litigious divorce, which neither principal wanted.

Levine v. Levine, 436 N.E.2d 476 (NY 1982) (*Levine*)

An attorney represented both Husband and Wife in a dissolution of marriage. After the dissolution, Wife sued to set aside the agreement on the grounds of the coercion and overreaching in the common agency. The court, upon reviewing the facts in detail, held that common agency in a dissolution is not always barred, and Wife consented to the common agency after full disclosure. Though Wife claimed Husband was doing much better than he disclosed in the court record, she provided no evidence.

As with *Klemm* the principals are in a Battle of the Sexes. Husband was in the retail auto supply business, one in which it may be relatively easy to conceal revenue. Wife had been the business bookkeeper so it may have been unlikely he was concealing it from her. From the assets mentioned in passing, Cadillacs, boats, houses, it is apparent that the Levines were doing better than the $20,000 per year disclosed to the court. Providing evidence of more income and assets would have helped third parties, like creditors or the IRS, shrink the bargaining surplus. Wife cannot prove after the fact that disclosure was incomplete, supposing it was, without risking an attack from third parties and endangering the surplus she seeks.

Type: θ_S, Battle of the Sexes. Levine is a the usual sort of two equilibrium game, but the amicable equilibrium is a little different. It is not that it saves fees, as in *Klemm*, or that the amicable route may be Pareto superior. Here, the marital pie was bigger if confidentiality from third parties could be preserved, but to make the case that she did not waive after full disclosure, Wife would have to disclose hidden assets to the court and so to third parties. She is stuck with an equilibrium *ex post* she does not prefer.

Efficient: Yes, to the extent verifiable.

Consistent: Yes. Wife could not disprove wavier after full disclosure.

Messing v. FDI, Inc., 439 F.Supp. 776 (1977) (_Messing_)

Braun, Gregg and Peltz ("Insiders") were directors of Filter Dynamics International, Inc. ("FDI"). Katten, Muchin, Gittles, Zavis & Galler ("Katten") was FDI's general counsel. Insiders and Katten controlled 42 % of the stock of Rayco International, Inc. ("Rayco") and guaranteed some of its debt; Katten was also Rayco's general counsel. An important asset of Rayco was its accumulated tax losses. FDI merged with Rayco, at far too high a price, it later turned out, because it was unable to take advantage of the tax losses. Stockholders of FDI sued FDI, Katten, the Insiders, all but two of the other directors of FDI ("Outsiders") and the investment bankers who opined that the merger was fair to FDI ("Bankers"). FDI was named as a defendant, but the stockholders sought to force the others to pay FDI in the event they were successful. FDI also filed claims of its own against Bankers. The Weston-Sills firms represented FDI, Insiders and Outsiders. Bankers objected that there were conflicts of interest between FDI and both Insiders and Outsiders, and conflicts between Insiders and Outsiders.

The court noted that FDI was an active participant in the case, having raised claims against Bankers, and stood to gain if Insiders and Outsiders lost. Also, FDI and Insiders and Outsiders were on opposite sides concerning advance of litigation costs by FDI to the Insiders and Outsiders. Separate counsel was required. The court considered how FDI might obtain independent counsel when almost all of its board members had conflicts. It referred to state law procedures in the event of conflicts of interest between directors and their corporation. Outsiders might have a defense of casting blame on Insiders, but if they agreed to common agency after full disclosure, the common agency was permitted.

Type: θ_P, Prisoner's Dilemma. In *Messing* all the parties except Bankers are trying to extract money from Bankers, and Bankers complains about conflicts among them. This is a Prisoner's Dilemma like *Kerry* or Stag Hunt like *Aetna.* On the other hand, FDI is a corporation all of whose management is embroiled in the litigation. It does not have a distinct will, even though it has a distinct role and the possibility of gain at the expense of the Insiders and Outsiders who control it. Until it has independence, it does not have the ability to choose a strategy. Compare *Hurt*, where the dependent principal is an infant. If FDI can be given a independent bargaining representative, it can knowledgeably agree to the arbitrator solution of the Prisoner's Dilemma.

Efficient: Yes. The court ruled that FDI required a decision-maker in the form of counsel, but did not prohibit FDI from then acting in common agency with Insiders and Outsiders.

Consistent: Yes.

***MGIC Ind. Corp. v. Weisman*, 803 F.2d 500 (9th Cir. 1986) (*MGIC*)**

Kersting was a stockholder and director of the First Savings and Loan Association of Honolulu ("Bank"). The officers and directors of the Bank were insured by MGIC. After the bank failed, Kersting and other stockholders, retained Weisman to threaten to sue the directors. The directors, through Weisman, then sued MGIC for a declaration that the insurance was in effect. Kersting, through Weisman, engaged in a campaign to get himself and other directors sued by third parties. Eventually, at Weisman's urging, FSLIC, the bank's regulator, and First Hawaiian Bank both sued the directors. Weisman defended the directors.

MGIC sued Weisman accusing him of various ethical violations, fraud and racketeering. The court held that there was nothing wrong with Weisman representing Kersting, despite Kersting's multiple roles as stockholder/plaintiff and director/defendant. There was nothing wrong, the court held, in the directors trying to get sued to trigger the insurance policy. Under the insurance policy, MGIC was obligated to pay for their legal defense. MGIC claimed that Weisman had conflicts of interest which he did not disclose to MGIC, but the common agency was obvious in the court records, and MGIC and its separate counsel knew it. Not only did MGIC lose its case over conflicts of interest, it was sanctioned for frivolously compounding its claims with accusations of racketeering and other crimes.

A stockholder/director is a single human being whose interests, however conflicted, can be represented by a single lawyer. They are not the lawyer's conflicts. More of a problem is acting in the stockholder interest while defending the directors *for the insurer.* The stockholders and MGIC are in a Matching Pennies Game θ_M, a direct conflict of interest. Offsetting this, at least in the court's analysis, was the character of MGIC. As an insurance company, sophistication in legal matters is presumed, and MGIC did have separate lawyers for some parts of the litigation.

Type: θ_M*, Matching Pennies.* Attorney Weisman represented the stockholder/directors advocating their claims as stockholders, and represented stockholder/directors defending their conduct as directors, with a duty of loyalty to the directors' insurer MGIC. The case has none of the flavor of the multiple equilibrium cases.

Efficient: No. This is a difficult case. Weisman could not be loyal to the stockholders-/directors as stockholders and stockholder/directors as directors at the same time. However, if they were the same persons, it is they that had the conflict, not he. On the other hand, in representing the directors he had a duty to their unconflicted insurer. Finally, however, the insurer had some degree of separate representation. In finding the case difficult, I disagree with the court, which found it easy.

Consistent: No. It is equally hard to reconcile the court's ruling with precedent. Weisman had a direct conflict of interest, which is usually not waivable.

***Sapienza v. New York News, Inc.*, 481 F.Supp. 676 (SDNY 1979) (*Sapienza*).**

Cohen was a newspaper distributor for the New York News ("News"). Cohen got into a dispute with the News over this distribution contract. Tarnow represented Cohen in that suit, which was settled by renegotiation of the contract. The renegotiated contract included provisions that barred Cohen's company from distributing for other papers. Guido Sapienza was a newspaper carrier who distributed the News through Cohen's company. The newspaper carriers ("Guido") sued the News, Cohen and others alleging that the contract provisions between the News and Cohen violated antitrust law. Tarnow represented Guido against the News and Cohen. Although Cohen was inactive as a defendant, Tarnow also represented Cohen.

It appears that Cohen was the "real" client. Guido had legal grounds to complain about the contract when Cohen, who had agreed to it, could not. Despite that fact that Guido was suing Cohen for treble damages, Guido and Cohen were not adversaries in fact. The court ruled that Tarnow could not represent both Defendant Cohen and Plaintiff Guido. The court relied on the process integrity interest. If Guido and Cohen were not really adversaries, then setting them up as adversaries was collusive and they should be realigned as Plaintiffs. Under the posture of the case, the court would lose jurisdiction and there would then be no case. If they were truly plaintiff and defendant then they must be adverse and common agency would undermine "the confidence and respect of the community toward its bench and bar."

Type: θ_1, *One-equilibrium.* Though Guido was Plaintiff and Cohen named as a Defendant, both represented by Tarnow, their interests were aligned, and there was one Pareto superior strategy for them to adopt. From their perspective it was a win/win game, Figure III.9.

Efficient: Yes. The court acknowledged that common agency was efficient here, but the commonality of interest made the litigation a fraud, a process integrity issue outside the scope of the game theory model.

Consistent: Yes. The process integrity value was implicated.

***Universal City Studios, Inc. v. Reimerdes*, 98 F.Supp.2d 449 (SDNY 2000) (*Universal*).**

See main text.

***Wait v. Second Judicial District Court*, 407 P.2d 912 (Nev. 1965) (*Wait*)**

Wait is another variation on the divorce theme. Attorney Wait represented Husband and Wife in a suit about a slip and fall in their grocery store. But Husband and Wife were feuding, and Husband threatened to give true but incriminating evidence about the accident. Wait sought to drop Husband as a client and continue representing Wife. Wait was ordered to withdraw entirely because of his conflict. On appeal the court held there was no conflict of interest, and so no need for complete withdrawal.

If the principals were rational they would act together to preserve the value of the marital estate. Here Husband favored a result out of spite. Given Husband's conduct, common agency was impractical, but Wait's representation did not harm Husband's financial interest.

Rather then resort to irrationality as a basis for decision, one may view the marriage as a repeated game between Husband and Wife. As before, informed consent to common agency should be required in a synergistic game, and was obviously absent here. On the other hand, the economic analysis does not clearly address the choice before the court: allowing Wife to continue the case with Wait, or disqualifying Wait. The analysis does not address whether disqualification of Wait should be available to Husband as a strategy to punish Wife. Allowing Wait to continue implicates Husband's loyalty interest, but, perhaps, not Husband's confidentiality interest. Hence the result is consistent with *Universal.*

Efficient: Yes. A One Equilibrium Game θ_1.

Consistent: Yes.

***Worldspan, L.P. v. The Sabre Group Holdings, Inc.*, 5 F.Supp.2d 1356 (ND Ga 1998) (*Worldspan*).**

This case presents, perhaps, the middle ground between *Dresser* and *Universal.* Plaintiffs Worldspan retained Georgia counsel to represent it in tax litigation in Tennessee and Georgia. Worldspan objected when it received a long form letter informing them that their lawyers were a large law firm with many clients, and that Worldspan waived common agency if the other representation was unrelated and did not use confidential information. Worldspan complained but was informed that the letter was not negotiable. Six years later, the firm was retained by the defendants to oppose Worldspan in tort litigation over airline reservation systems. Worldspan objected.

The court stated that the key attorneys in the tort litigation were national law firms on both sides. Georgia counsel's role was not crucial and it was replaceable. On the other hand, the court opined that tax lawyers gain familiarity with many aspects of the business of their clients, and the firm may have gained insights it could use against Worldspan now. The court found no indicia that the motion was a tactical stunt like the one in *Universal.* Here the confidentiality issue was the determinative one. Georgia counsel was disqualified.

Type: θ_M, Matching Pennies. Worldspan's tax lawyers are suing it in a tort claim.

Efficient: Yes. As with some of the other Matching Pennies Games θ_M, the question is whether the two cases at issue are so related that representation in one commits the agent with respect to the other. The court found strategic interaction and impairment of the confidentiality interest and so disallowed common agency.

Consistent: Yes. Confidentiality interest.

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